Quantitative RMA For The Practitioner

(For Self-Study Using Microsoft Excel)

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One "Take Away"

Good **D**ecisions = **T**ime * **I**nformation

Good information is a function of good data

- This presentation will illustrate the benefit provided by time-based data and the use of Microsoft Excel.
- With the correct data and math model, RMA specialists can help organizations make risk-informed decisions.
- The next page provides details on the technical content.

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Technical Content

Prerequisites

1. [**p. 4**] Make a histogram from sample data and determine the histogram's best fitting probability density function (PDF).
2. [click here] Make and recognize the graphs for and describe the relationship between the following four math models based on the Weibull and lognormal distributions: PDF, cumulative distribution function (CDF), complement of the CDF, and ratio of the PDF and complement of the CDF.

Goal

1. [**pp. 5 & 6**] Answer common management questions pertaining to quantitative engineering assurance that require the:
   a. Determination of probability values as point estimates (not interval estimates) for reliability, maintainability, and availability (RMA) using Microsoft Excel, and
   b. Knowledge of selected conceptual underpinnings in order to plan, perform, communicate, and defend the work.

Instructional Objectives

1. [**pp. 8-10**] Using data, calculate maintainability based on the lognormal probability distribution.
2. Using parameters, calculate reliability based on the Weibull probability distribution* for three cases, namely:
   a. [**p. 11**] New item
   b. [**p. 14**] Used item (conditional reliability), and
   c. [**pp. 15 & 16**] New item under idealized maintenance.
3. [**p. 12**] List three common sources for life data, the data used to make the parameter(s) for the reliability math model.
4. [**p. 13**] Using means (averages), calculate inherent (design) steady-state (limiting, asymptotic) availability and not inherent point (instantaneous) availability and not inherent average (interval, mission) availability.
5. [**pp. 18-21**] Describe how the Weibull shape parameter (β) influences reliability under the strategies of break-in (burn-in, wear-in) and preventive maintenance.
6. [**pp. 23-28**] Describe conceptually the major steps or work process used to convert data (i.e., time to failure and time to non-failure, time to repair) to a math model (i.e., reliability, maintainability).

*The Weibull probability distribution models time-based (continuous) data and not demand-based (discrete) data as with the binomial and Poisson probability distributions.

Core Concepts for Probabilistic Models

- **Sample Data**: A collection of measures (e.g., time to failure, time to non-failure, time to complete a repair) for items of the same type where each data entry is associated with a unique item. Thus, each measure is associated with a unique identifier such as the item's serial number.
- **Histogram**: A graph that shows the frequency of the data (collection of values) in a particular time interval (bin). To make a histogram, divide the entire range of values into a series of intervals, and then count how many values fall into each interval. The intervals make the horizontal axis, and the counts (frequencies) make the vertical axis. Microsoft Excel's "Analysis ToolPak" add-in makes histograms.
- **Probability Density Function** (PDF) is a plot of a theoretical mathematical function or equation that approximates the shape of the histogram. The assignment of a particular probability distribution is by inspection or by a software tool (i.e., curve fit, distribution fit).

What is RMA?

- **Reliability** (R) is the probability an item will be in an **uptime** state (i.e., fully operational or ready to perform) for a stated mission under stated conditions.
- **Maintainability** (M) is the probability an item in a **downtime** state will be returned to an uptime state within a given period of time.
- **Availability** (A) is a function of Reliability and Maintainability (R&M) and is the probability a repairable item will perform its intended function at a given point in time (or over a specified period of time) when operated and maintained in a prescribed manner.
- First, design for the reliability (**R**). Second, if reliability is not sufficient, then design for maintainability (**M**). When an item entails both R and M, the performance metric is availability (**A**), a mathematical function of R and M.

Central Questions of Probabilistic RMA

What is the probability that a:

1. **M**: Repair on an item will be completed (end a downtime state in order to return to an uptime state) on or before a stated time?
2. **R**: New item will complete its mission with no downtime?
3. **A**: Repairable item over the long term (steady state) will be in an uptime state? Note: Mission time approaches a large value.
4. **R**: Used item will complete its mission with no downtime?
5. **R**: New item will complete its mission with scheduled downtime using ideal preventive maintenance?

Microsoft Excel Examples

For Each RMA Central Question



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Question 1: $M(t_i)$ = ?

- What is the probability an item will be repaired in 2 hours (t_i = 120 minutes)? The item's repair-time history and frequency distribution of the repair times (histogram) are provided on the next two pages.

Answer:

- Since it is postulated the repair history fits a lognormal probability density function (PDF), transform each repair time with the natural logarithm [i.e., $t \rightarrow \ln(t)$]. For this transformed set, the mean (μ_n) is 5.295134 minutes and the population standard deviation (σ_n)* is 0.653912 minutes.

- $M(t_i)$ = LOGNORM.DIST(t_i, μ_n, σ_n, True)

  M(120) =LOGNORM.DIST(120,5.295134,0.653912,True) = 0.2188 ≈ **22%**

- Thus, based on the history, as a point estimate there is a 22% chance the item will be repaired in 2 or less hours (or 78% chance after 2 hours).

* Note: The population standard deviation (not the sample standard deviation) is used when the maximum likelihood estimator (MLE) is used with complete data (no censored times) to estimate the parameters of the lognormal probability distribution.

75 historical times (minutes) to repair item x

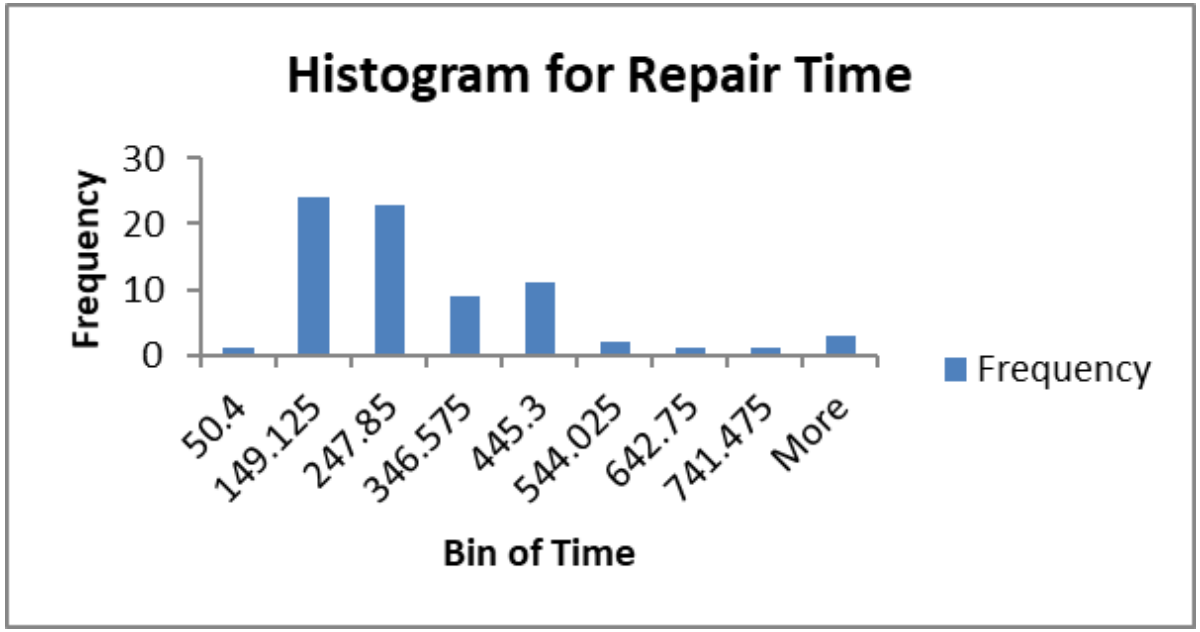
50.4	92.2	115.3	178.4	216.4	253.2	371.4	559.1
56.2	96.1	122.7	184.5	222.6	263.6	382.1	643.8
72.6	99.7	128.3	187.2	231	273.3	383.5	789.3
73.3	102.5	131.1	189.7	231.4	295.1	385	830.7
76.1	103.7	141.3	193.4	237.3	305.2	414	840.2
78.5	104.8	166	203.5	238.6	310.4	420.5	
80.6	105	166.1	204.1	243.7	340.7	426.5	
83.4	106.8	168	204.4	244.7	349.4	431	
84.6	107.3	170.6	215.3	252.1	355.8	457.4	
89	109.2	174.4	215.8	252.2	363.6	462.9	

- Statistics for **untransformed data**: Median = 204.40, mean (average) time to repair (MTTR) = 246.80, and sample standard deviation = 173.24.
- Statistics for **transformed data**: Median = 5.326816, mean (average) time to repair (MTTR) = 5.295134, sample standard deviation = 0.658316, and population standard deviation = 0.653912.
- Statistics via **lognormal relationships** based on transformed data: Median = 199.36 via =EXP(μ_n) or =LOGNORM.INV(0.5,μ_n,σ_n) and mean = 246.89 via =EXP(μ_n+0.5*(σ_n^2)).



75 repairs times as a histogram

- The **histogram** below is made by Excel's "Analysis ToolPak" add-in.
- The shape of this histogram suggests the lognormal probability distribution's probability density function (PDF).
- The **lognormal PDF** "is often used to model usage data, such as … repair time of a maintained system." (Practical Reliability Engineering, 5th Edition, O'Connor, 2012, pp. 35 & 410)



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Question 2: $R(t_i)$ = ?

- What is the probability a **new item** will operate successfully with no downtime for 800 hours (i.e., from 0 to 800)? The vendor will not provide the failure history, but the vendor did describe the life data as a Weibull probability distribution with a shape β^* = 3 and scale θ^* = 1000 hours.

Answer:

- $R(t_i) = Exp\{-[t_i/\theta]^\beta\}$ = Excel's "=1-WEIBULL(t_i, β, θ, True)"

  R(800) =1-WEIBULL(800,3,1000,True) = **0.5993**

- Mean Time To Failure (MTTF) =1000*GAMMA(1+(1/3)) = **893.0 hours**

Note:

- For the Weibull model, $R(\theta) = e^{-1} \approx 0.367879$ regardless the value of the shape parameter β. Other words, **when mission time t_i is the same value as the scale parameter θ**, all Weibull (and exponential) models equal to e^{-1}. Recall, e ≈ (1+1/n)^n for large n or $e = \sum_{n=0}^{\infty} 1/n!$

*Click here for methods to estimate parameters for a probability distribution.

Sources for Life Data

In general, **life data has three sources**: Your database, parameters from a vendor (as in Question 2) or handbook (e.g., NPRD), or expert opinion (formally called Expert Elicitation).

To forecast an uptime state (life) for a new item when you have no failure history:

- Ask the vendor for the life parameters. In general, there are **two basic statistical parameters**--and both are needed, namely:
  - Measure of central tendency (center of the data), e.g., mean, median, mode.
  - Measure of variability (spread of the data), e.g., standard deviation, error factor, scale.
- Start **your own data collection program** even if your vendors provide the above. Your data collection gives each item a unique identifier (e.g., serial number) and the following:
  - Date of installation (or start of operation).
  - Age (e.g., previous run time) if the item is not new.
  - Date of each failure or removal from service.
  - Duration of operation (e.g., run time) between each failure.
  - Duration of operation (e.g., run time) for items in a non-failed state.
  - Duration of the downtime state.
  - Reason for each failure (e.g., failure mode and/or failure mechanism).
  - State if the item's return to service (fix) was due to a repair or replacement. Note: If item is new, assign a unique identifier to this new item.

Tip: To provide a reliability measure when the accumulated run time contains no failures, use the **method for zero failures**. See the "Tools" section at the "KSC Reliability" web site.

Question 3: A = f(R,M)

- What is the probability an item over the long run will be in an uptime state (ready for use or in use)? The modeled data has average repair time (MTTR) equal to 246.89 minutes (4.1 hours) and average life time (MTTF, replaced and not repaired) equal to 893.0 hours (ref. Questions 1 and 2).
- Forecasting uptime with allowable downtime is **availability**. When the mission time is a long-time interval (as opposed to a point or a short-time interval relatively close to zero), this type of availability is called **Steady-State (Limiting, Asymptotic) Availability**, A_∞.

Answer:

- A_∞ = (Average Uptime)/(Average Uptime + Average Downtime)

  A_∞ = 893.0/(893.0 + 4.1) = 893/897.1 = **0.9954**

- Thus, over the long run this item will be available approximately 99.5% of the time and not available 0.5% of the time.
- For a brief overview on the various types of Availability, click here

Question 4: $R(t_i|T_0)$ = ?

- What is the probability a **used item** will operate successfully with no downtime for 800 hours given that it has successfully operated 100 hours? Assume this item has the following life parameters: Weibull math model, shape parameter β = 3, and scale parameter θ = 1000 hours (ref. Same as Question 2).

- Note: In Question 2, the item is new and has a 800-hour mission. In this question, the item is used and also has a 800-hour mission duration. However, the used item's total (cumulative) run time upon completion of 800-hour mission will be 900 hours.

Answer:

- Use **conditional reliability**. In general, this is: $R(t_i|T_0) = R(T_0+t_i)/R(T_0)$ where $R(t_i|T_0)$ denotes "R at t_i given that it has successfully operated to time T_0."

- **Weibull conditional reliability**: $R(t_i|T_0) = Exp\{-[(T_0+t_i)/\theta]^\beta\}/Exp\{-[T_0/\theta]^\beta\}$

- R(800|100) =(1-WEIBULL.DIST(100+800,3,1000,True))/(1-WEIBULL.DIST(100,3,1000,True)) = **0.4829**

Note: The Weibull with β = 1 is the same as the **exponential probability distribution**. In these cases, previous exposure time (age) and stresses are not remembered. Observe at β = 1: R(800|100) = R(800|0) = Exp(-800/1000) = 0.4493



Question 5: $R_M(t_i)$ = ?

- What is the probability a new item with idealized preventive maintenance (PM) performed every 500 hours will complete a 800-hour mission? Idealized PM means the item is restored to a condition "as-good-as-new." This item has the same life math model as in Question 2 (i.e., Weibull with shape $\beta = 3$ and scale $\theta = 1000$ hours).

Answer:

- Use the **reliability under idealized maintenance model**, $R_M(t)$. In general, this model is: $R_M(t_i) = R(T)^N R(t_i - NT)$ where t_i is the operation or mission time, T is the length of the PM interval, and N is the number of PM intervals or cycles performed on or before the completion of mission time t_i. Thus, $NT \leq t_i < (N + 1)T$. For this question, $N = t_i/T = 800/500 \rightarrow$ use 1.

- When the reliability model is the Weibull distribution, the reliability under ideal PM model is: $R_M(t_i) = \text{Exp}[-N(T/\theta)^\beta]*\text{Exp}[-((t_i - NT)/\theta)^\beta]$

- $R_M(800)$ =((1-WEIBULL.DIST(500,3,1000,TRUE))^1)*(1-WEIBULL.DIST(800-(1*500),3,1000,TRUE)) = **0.8590**

(continued on next page)

Question 5: $R_M(t_i)$ = ? (continued)

- Observe the probability or **reliability of 0.5993** in Question 2 (i.e., a new item completes an 800-hour mission with no downtime or failure) **increases to 0.8590** in this question (same as Question 2 except the item completes the mission in 500-hour intervals **with idealized preventive maintenance** performed at the end of each interval).
- In regards to measuring **the benefit of idealized PM** performed on this item, a comparison of the failure probabilities (un-reliabilities) indicates the failure probability without maintenance is 2.8 times larger than the failure probability with maintenance.
- Note: Since this item has an increasing failure rate over time (i.e., Weibull's $\beta > 1$), preventive maintenance increases reliability. Preventive maintenance decreases reliability when the failure rate decreases over time (i.e., Weibull's $\beta < 1$). Preventive maintenance will not change reliability when the failure rate remains constant over time (i.e., Weibull's $\beta = 1$).

Applied Reliability Principles



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Reliability vs. Math Model and Failure-Rate Trend

- Recall, each of the previous reliability questions used the Weibull distribution with a shape parameter $\beta = 3$, scale parameter $\theta = 1000$ hours, and mission time $t_i = 800$ hours.
- In the table, observe the changes in the probability measure by model type and by failure-rate trend type. These trends are the Weibull instantaneous failure (hazard) rate being a function β, θ, and t_i.

Question & Reliability Model	Illustrated Weibull Instantaneous Failure Rates		
	Decreasing, β=0.5	Constant, β=1.0	Increasing, β=3
Q2: new item (age = 0 hrs.)	0.4088	0.4493	0.5993
Q4: used item (age = 100 hrs.)	0.5313	0.4493	0.4829
Q5: new item with idealized PM every 500 hrs.	0.2851	0.4493	0.8590

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Weibull Shape Parameter vs. Break-In

- **Conditional reliability** (ref. Question 4) is useful in describing the reliability of an item following a break-in (burn-in, wear-in) period. Conditional reliability's T_0 can be viewed as the time interval for an item's break-in period before being placed into service. However …
- A **break-in period** benefits reliability if the instantaneous failure rate is decreasing [i.e., $\lambda(T_0) > \lambda(T_0 + t_i)$]. With a Weibull model, the failure rate decreases over time when its shape parameter is less than one ($\beta < 1$).
- When the failure rate is constant over time ($\beta = 1$), a break-in period has no benefit since previous failures are not remembered.
- If the failure rate increases over time ($\beta > 1$), a break-in period only adds to the deterioration of the item. That is, the reliability during the mission time of interest (i.e., incremental time period t_i) decreases due to run time during the break-in period.

Weibull Shape Parameter vs. Preventive Maintenance

- As stated at the end of Question 5, **preventive maintenance**:
  - Increases reliability when an item has an increasing failure rate over time (i.e., Weibull's $\beta > 1$).
  - Decreases reliability when the failure rate decreases over time (i.e., Weibull's $\beta < 1$).
  - Will not change reliability when the failure rate remains constant over time (i.e., Weibull's $\beta = 1$).

When the Weibull Shape Parameter β = 1

- When the Weibull shape parameter β equals 1.00 and the Weibull scale parameter θ is the same value in each of the following methods, then reliability for a new item, conditional reliability for a used item, and reliability for a new item under idealized preventive maintenance methods yield the same reliability measure.
- Other words, when the Weibull β = 1.00 and the Weibull θ is the same for each method, then $R(t_i) = R(t_i|T_0) = R_M(t_i)$. These three methods simplify to $Exp(-t_i/\theta)$. In addition, $Exp(-t_i/\theta) = Exp(-\lambda * t_i)$ since $1/\theta = \lambda$, a common notation for a constant failure rate.
- In addition, when β = 1 and as $t_i \rightarrow \theta$, then $R(t_i) \rightarrow e^{-1} \approx 0.3679$. These values for β and t_i make a failure probability (unreliability) approximately equal to 63%. This is the reason why a high reliability measure requires a high MTBF when λ is a constant.

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Process:
From Data To R&M Model



General Process: From Data to Math Model

1. Obtain data (e.g., time to failure, time to repair).
2. Make histogram for the data. (ref. Question 1)
3. Postulate the probability density function (PDF) for the histogram.
4. Select estimator method (e.g., regression, maximum likelihood estimator, or do both to provide a graphical plot and best fit).
5. Use the estimator with the data to estimate the unknown parameter(s) of the postulated PDF.
6. Determine the goodness of fit of the data and PDF (optional).
7. Use parameters to make the math model of interest. In Excel, the complement of the Weibull and Lognormal's Cumulative Distribution Function (CDF) is Reliability. That is, R = 1 – CDF.
8. Use the math model to calculate a point estimate.
9. Describe statistical uncertainty (i.e., interval estimate via confidence interval or probabilistic simulation).



Functional Relationships in Maintainability

1. The math model that fits the time to repair (TTR) histogram is the **Probability Density Function** (PDF).
   - Denoted f(t) or m(t).
2. The cumulative sum of the PDF from left to right is the **Cumulative Distribution Function** (CDF) being **Maintainability**.
   - Denoted $M(t) = \int m(t)dt$ from 0 to t_i where t_i is the repair time of interest.
3. The ratio of the PDF and the complement of Maintainability is **Instantaneous Repair Rate**.
   - Denoted $v(t) = m(t)/[1 - M(t)]$.

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Making the Maintainability Math Model

- Time to repair (TTR) data for every item of interest → Obtain t_1 to t_k being all or part of available TTR data → Make histogram → Assume the data fits lognormal distribution if histogram is unimodal and skewed to the right → Convert (transform) each repair time to its natural logarithm → Use the transformed repair times to estimate the parameters of the lognormal distribution being the mean (μ_n) and population standard deviation (σ_n) → Make unique lognormal model using the parameters → For a stated repair time (t_i), calculate maintainability being a point estimate → Result: **$M(t_i) = Pr\{ T \leq t_i \}$**, probability the repair will be completed in t_i or less amount of time.

- **Excel Formula for $M(t_i)$** =LOGNORM.DIST(t_i,μ_n,σ_n,True) where μ_n =AVERAGE(Ln(t_1):Ln(t_k)) and σ_n =STDEV.P(Ln(t_1):Ln(t_k))

Functional Relationships in Reliability

1. The math model that fits the time to failure (TTF) histogram is the **Probability Density Function** (PDF), denoted f(t).

2. The cumulative sum of the PDF from left to right is the **Cumulative Distribution Function** (CDF), denoted $F(t) = \int f(t)dt$ from 0 to t_i where t_i is mission time.

3. The complement of the CDF is **Reliability** (Survival Function), denoted R(t) or S(t). $R(t) = 1 - F(t) = R(t) = \int f(t)dt$ from t_i to ∞.

4. The ratio of the PDF and reliability is **Instantaneous Failure (Hazard) Rate**, denoted $\lambda(t)$ or h(t). $\lambda(t) = f(t)/[1 - F(t)] = f(t)/R(t)$.

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Making the Reliability Math Model

- In theory, there is a time* to failure (TTF) and non-failure time* data set for every item of interest → Obtain all or part of the available failure and non-failure time data → Make histogram → Typically life data fits a Weibull distribution if histogram is unimodal → Use software tool to estimate the Weibull's shape (β) and scale (θ) parameters → Make unique Weibull model using the parameters → For a stated mission time (t_i), calculate reliability being a point estimate → Result: **$R(t_i) = Pr\{ T > t_i \}$**, the probability the item will perform its intended function more than t_i amount of time.
- **Excel Formula for $R(t_i)$** =1-WEIBULL(t_i,β,θ,True) which solves $R(t_i) = Exp[-(t_i/\theta)^{\wedge}\beta]$
- **Mean of the Weibull** =θ*GAMMA(1+(1/β)) which is not same as the average of the times to failure.

* Note: Input data that builds a reliability math model is not limited to time-based (continuous) data. Other reliability data types are demand-based (discrete) data and stress-strength (load-capacity) data.

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Tips: Using the Weibull Math Model

- When using Excel, do not reverse the arguments for β and θ!
- The standard Weibull is for the time interval from zero to t_i and assumes the item is new. When the item is non-repairable and not new, use the conditional form of the Weibull.
- When the failure rate (λ) is assumed to be constant over time, then β = 1 and θ = 1/λ, where θ is the mean time to failure (MTTF) or mean time between failure (MTBF). MTTF and MTBF are the same value when a failed item is restored to a as-good-as new condition. Otherwise, for a repairable item, the MTBF will decrease over time.
- **When β = 1, the Weibull model indicates** failures are due to random stresses (loads) beyond the intent (strength or capability) of the item's design. In this case, preventive maintenance provides no benefit to reliability. Downtime for this reason is corrected by repair, change in environment, design change, or any combination.