

How to do Quantitative Reliability

- Concepts, Motivation, and Methods to Find the Probability of Failure -

Timothy C. Adams
NASA Kennedy Space Center
Engineering
Tim.Adams@NASA.gov

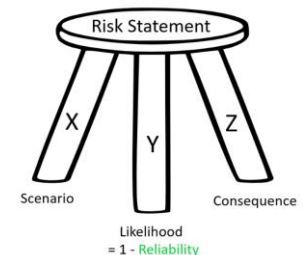
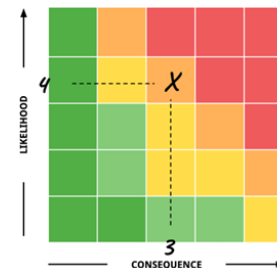
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Prospectus

- ◆ Claim: To make good [risk-informed decisions](#), both axes (likelihood and consequence) of the risk matrix need to be properly analyzed and described.
- ◆ Goal: Motivate practitioners to ...
 - Estimate, analyze, describe, and discuss the likelihood portion of a **risk statement** using the probabilistic method *in addition to* the deterministic method.
 - Communicate and advocate: Operational (demonstrated) data ([slides 5, 6, 14, and 20](#)) and not technical data is the **essential resource** used to make the probability portion of a reliability statement. Organizations that collect and use operational data *reduce the number of inferences* used to make credible forecasts for quantitative reliability (also an inference).
- ◆ What & Why:
 - What follows are selected concepts, methods, and examples from the Probability and Statistics discipline that support decisions under risk and uncertainty, specifically, p_f .
 - Why: **Actual Results = Planned Results +/- Risk.**
 - In failure space, risk is *potential loss*.
 - “Potential” is the “Likelihood” axis of the risk matrix.
 - The likelihood axis is the probability of failure (p_f).
 - $p_f = 1 - \text{Reliability}$.
 - As a measure for p_f , Reliability uses Probability and Statistics.



Risk: Reliability Makes the Likelihood Component

Risk as a Concept

- ◆ Risk is:
 - Potential loss **or** potential gain.
 - The uncertain deviation (delta) in the execution of a management plan.
 - Resources:
 - [ISO 31000, Risk Management](#)
 - [NPR 8000.4C, NASA Risk Management](#)
- ◆ Risk when limited to potential loss (failure space) is:
 - A qualitative or quantitative estimate of the potential loss occurring due to natural or human activities.

Risk as an Operation

- ◆ Using both potential loss and gain:
Actual results = Planned results +/- Risk
- ◆ **Risk statement** limited to potential loss:
 - X, Scenario: What can go wrong?
 - Y, Likelihood: What is the probability it will happen ($p_f = 1 - \text{Reliability}$)?
 - Z, Consequence: What is the impact if it did happen? For example, loss of ...
- ◆ **Risk measure** = **Y*Z** = (Likelihood)*(Consequence) being a:
 - Number (i.e., product) or
 - Graphic (i.e., point or cell in a risk matrix).

Risk Likelihood (Y) = Probability of Failure (p_f) = Unreliability (U) = 1 - Probability of Success (p_s) = 1 - Reliability (**R**)

Risk Decisions (Handling, Responding): Accept (**f**ight), Avoid (**f**light), Hold (**f**reeze), Mitigate (**c**hange), and Transfer (**s**hare)

Definitions: **R** (↑) and its Counterparts **M** (↓) and **A** (↑ ↓)

Reliability (R)

- The probability an item¹ will perform its intended function with **no failure** for a given time interval and under given conditions (e.g., environment and loads).
- The probability an item will be in and remain in an uptime state.

Maintainability (M)

- The probability a **failed item**¹ will be restored or repaired to a specified condition within a given time interval.
- The probability a failed item (an item in a downtime state) will be returned to an uptime state within a given time interval.

Availability (A)²

- The probability a **repairable item**¹ will perform its intended function at a given point or interval of time when operated and maintained in a prescribed manner.
- The probability a repairable item will be in an uptime state and will recover from downtime states.
- A mathematical function of R and M, $A = f(R, M)$.

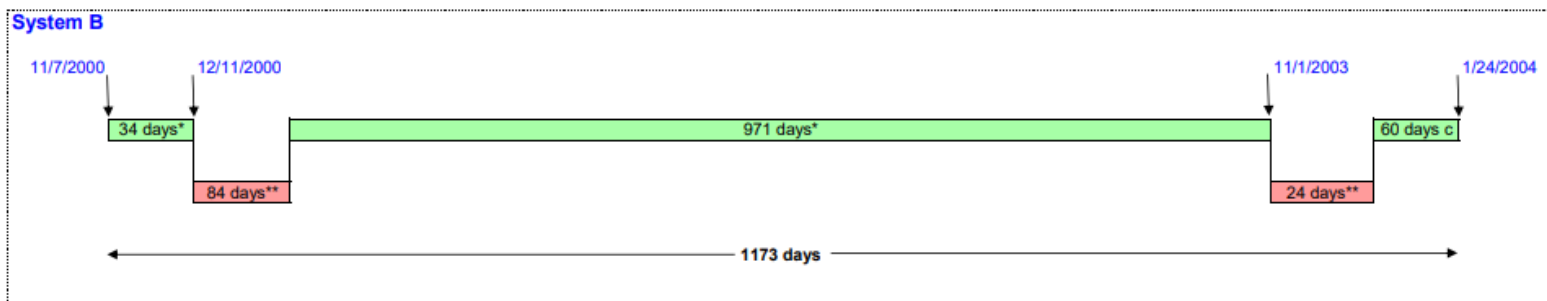
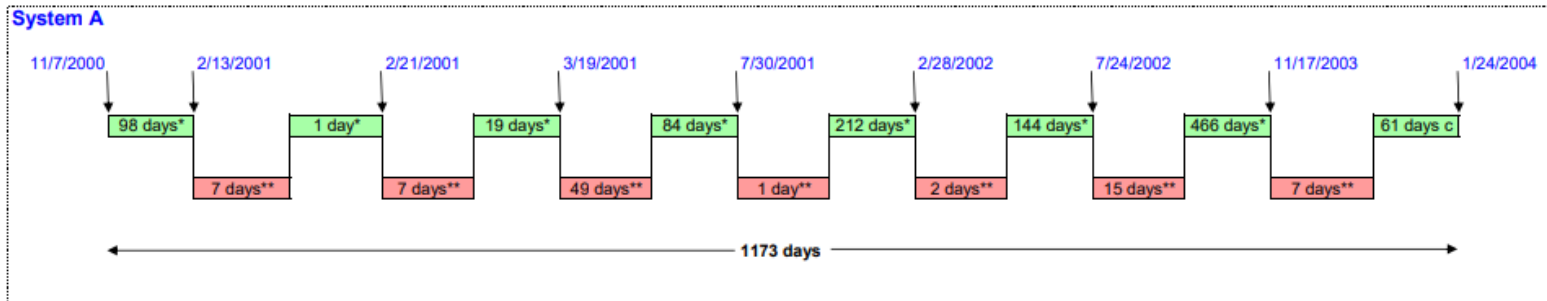
¹ An **item** is hardware (Hw), software (Sw), orgware (Ow) or humans, interfaces, or any combination.

² **System Availability** = $A_{\text{Hardware}} * A_{\text{Software}} * A_{\text{Orgware}} * A_{\text{Interfaces}}$ where “*” denotes “and” not “or.”

System Effectiveness is: (1) System Availability, (2) Dependability (i.e., operating condition, trustworthiness), and (3) Capability (i.e., meets mission demands).

Example: **R** and **M** Time-Based Data Used to Infer **A**

Uptime-Downtime Charts for Actual Hardware



Notes:

* denotes Uptime or operated ** denotes Downtime or under repair c denotes still operating Charts are not to the scale

This example uses basic **operational data**. A comprehensive data collection program collects other data elements. For example, a known Problem Reporting and Corrective Action (PRACA) system contains the following data elements:

“**Part number x/part name x/serial number x** failed during **Test/Operation x** while exposed to **Prevailing Condition x**. **Failure Mode x** was isolated to **Material x** being **Defect x** due to **Cause x**. The **Recurrence Control** (no action/remedial/corrective action) was x.”

Tip & Opportunity

- Use Probability and Statistics when the data contains uncertainty -
- Uncertainty is common in the data from both the operational and technical communities -

- ◆ To convert uptime and downtime data **to information**, use the craft of Probability and Statistics.
 - **Operational data** (previous slide) is collected and released by management (and typically reported by the news media). **Technical data** comes from Engineering (and from accident investigations – obviously late). Both data types use (should use) Probability and Statistics to make information. Slides 5, 14, and 20 provide additional information on these two data types.
- ◆ Why? Because these data lack certainty.
 - Reporting to the customer “it probably will do this or that” uses probabilistic thinking.
- ◆ Our competencies and work processes need to be both:
 - Deterministic and Probabilistic. Start with deterministics; end with probabilistics.
 - The next two slides define Determinism, Probabilism, and Uncertainty.
- ◆ From the Internet ...
 - “**Uncertainty** changes with acquisition or loss of knowledge, **probability** changes with a change in the system we are trying to predict. You can think of probability as being the degree to which we still cannot predict some variable's value if we had all reasonable data at our disposal.”

Source: ["Probability and uncertainty," David Vose, Apr 15, 2015](#)

Thinking Probabilistically: What does that mean?

Deterministic versus Probabilistic

Deterministic: All data is known beforehand

- ▶ Once you start the system, you know exactly what is going to happen.
- ▶ *Example.* Predicting the amount of money in a bank account.
 - ▶ If you know the initial deposit, and the interest rate, then:
 - ▶ You can determine the amount in the account after one year.

Probabilistic: Element of chance is involved

- ▶ You know the likelihood that something will happen, but you don't know when it will happen.
- ▶ *Example.* Roll a die until it comes up '5'.
 - ▶ Know that in each roll, a '5' will come up with probability $1/6$.
 - ▶ Don't know exactly when, but we can predict well.

Source:

[The City University of New York, Queens College,](#)
[Christopher Hanusa](#)

- ◆ **Determinism** is the belief that every event, act, and decision is the inevitable consequence of antecedents (past events) that are independent of the human will.
- ◆ **Probabilism** is the belief that probability (likelihood) is the adequate basis for belief and action, since certainty in knowledge cannot be attained.

Source: [KSC Reliability](#) website, [Probability & Verification, Concepts & Principles](#)

Uncertainty: What is it?

- ◆ **Uncertainty** does not imply no knowledge, but it does imply the exact outcome is not completely predictable. As a measure, uncertainty is the “goodness” of an estimate.
 - Most observable phenomena contain a certain amount of uncertainty. **For example**, repeated measurements of physical items generate multiple outcomes with some outcomes more frequent than others.
 - **For example**, if several “**identical**” **specimens** of a steel bar in a laboratory were loaded for a pull test until failure, each specimen would fail at **different load values**. The load capacity of the bar is therefore a random quantity and is formally called in mathematics a **random variable**.
- ◆ Traditionally, uncertain design parameters were approached by using deterministic methods such as safety factors and safety margins.
 - Typically, **safety factors** and safety margins are derived from past experience, **ignore the variability** present in both the loads placed on a system and the system’s ability to react to the load, and do not absolutely guarantee safety or satisfactory performance.
- ◆ Four Types of Uncertainty: [* = especially applicable in the analysis of data]
 1. **Aleatory, Intrinsic, or Inherent** Uncertainty (e.g., as described above in the 1st bullet).
 2. **Parameter or Statistical** Uncertainty* (e.g., small samples or experts’ judgment with no data).
 3. **Model** Uncertainty (e.g., no consensus on approach or model).
 4. **Completeness** Uncertainty (i.e., limitations in the scope of the model and where the true risk lies).

Probabilistics & Uncertainty: How important is it?

- ◆ Probabilistic-based products benefit your customers.
 - This presentation will share the methodology for selected examples.
- ◆ I am not the only fan for thinking and working with Probabilistics.
 - **For example**, what follows was written over 20 years ago.
 - “The application of risk and reliability in the analysis, design, and planning of engineering systems has received worldwide acceptance from the engineering profession. As a result ... design guidelines and codes are being modified to incorporate the concept of **risk-based analysis and design**. The [Accreditation Board of Engineering and Technology \(ABET\)](#) now **requires all civil engineering undergraduates in the U.S.** to demonstrate knowledge of the application of probability and statistics to engineering problems ...”
Source: Probability, Reliability and Statistical Methods in Engineering Design, Haldar & Mahadevan, Wiley, 2000, p. xiii (Preface)
- ◆ Regardless what others say, it is wise to solve a problem using a variety of methods. Remember ...
 - “What if you are wrong?” Robert Conway, NASA Safety Center

Probability & Statistics: Basic Concepts (jargon)

- ◆ **Probability** as a discipline makes a conclusion about data from a sample based on the analyst's knowledge of the population. A **population** is the complete and entire collection of data on the system under study. A **sample** is a subset of the population.
- ◆ **Statistics** as a discipline makes a conclusion about the population based on the analyst's knowledge of the sample. As with verifying requirements, inferring certain facts about the population (flight hardware) from the results found in a sample (test hardware) drawn from that population uses a process called **statistical inference** (i.e., Hypothesis Testing).
- ◆ A **statistic** is a numerical measurement that describes some characteristic of a sample. A **probability** as a measure is the likelihood that some given event will occur and is assigned a value between 0 and 1. In practice, this number has to be estimated. As a best guess (not best case, not most optimistic), the probability is stated as a number and is technically called a **point estimate**.
- ◆ A **parameter** is a numerical measurement that describes some characteristic (e.g., mean) of the population. Thus, when only sample data is used, the parameter is most likely not the **true value** for the parameter and is only an estimate.
- ◆ The uncertainty encountered due to the math model's parameter(s) is described by an interval and is technically referred to as an **interval estimate** or as a **confidence interval**.
- ◆ A **histogram** is a graph that shows the frequency of the data in a particular interval. The histogram divides the range of values into intervals and assigns how many values fall into each interval. The intervals make the horizontal axis, and the counts (frequencies) make the vertical axis.
- ◆ **Probability Density Function (PDF)** is a plot of a theoretical mathematical function or equation that approximates the shape of the histogram. The assignment of a particular probability distribution is by inspection (i.e., postulated by expert opinion) or by a software tool (i.e., curve fit, distribution fit).

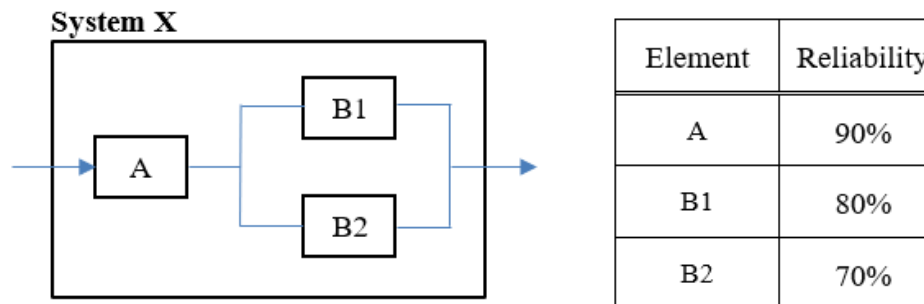
Probability: Labels, Laws, and Levels

- ◆ Probability of Success (p_s) + Probability of Failure (p_f) = 1.
 - Commonly written as: $p_s + p_f = 1$.
- ◆ Other crafts use this law with different labels. For example, ...
 - Reliability + Unreliability = 1. Medical community uses survival instead of reliability.
 - Availability (probability of uptime) + Unavailability (probability of downtime) = 1.
- ◆ Probability has no units of measure; Parameters do have units of measure.
 - For example, failure rate (λ) is a parameter; λ is commonly reported as failures per hour.
 - Tip: Ensure calculations use the same unit of measurement.
- ◆ When two or more items are under study, the probability of these items as a system (i.e., at the system level and not at the component level) is calculated using:
 - The [laws of probability](#) either manually or with software (e.g., [PTC Windchill Risk and Reliability](#) and [Relyence Software](#)). For NASA KSC, use [click here](#) [NASA internal].
 - For example, to manually calculate the probability of the reliability of a simple system, see the [next two slides](#).
 - Tip: A system-level probability requires two inputs: (1) Configuration of the items and (2) Probability of success or failure for each item (block) used in the system.

Find pf: At the system level (multiple probabilities) (part 1 of 2)

◆ Given

- Objective: Find the reliability (probability of success) for System X.
- Configuration: As shown in the diagram, System X has two items in series; the second item has two items in parallel. All items operate independently of each other.
 - **Independence** means the occurrence of success or failure in any one of the elements does not affect the probabilities of the occurrences of the other events.
- Selected Probability Laws:
 - Two items in series: Probability of A and B = $P(A \text{ and } B) = P(A) * P(B)$.
 - Two items in parallel: Probability of B1 or B2 = $P(B1 \text{ or } B2) = 1 - [1 - P(B1)] * [1 - P(B2)]$.



- Data (3 types): (1) The *reliability for each element* (block). (2) The likelihood System X will be needed, the *initiating event*, is one. (3) The *consequence* (e.g., loss of life, loss of property, additional cost, delayed schedule, loss of reputation) for failure is quantitatively unknown.

Find pf: At the system level (multiple probabilities) (part 2 of 2)

♦ Solution

- Write Outcome Statement (in English): Let S denote success and S' denote failure for “not S” or the complement of success. Based on the configuration of System X, system reliability is $P(S) = P[A \text{ and } (B1 \text{ or } B2)]$.
- Method 1 - Solve via Event Tree: (1) A, B1, and B2 generate eight (2^3) possible scenarios. Scenarios need to be assessed for applicability. (2) B is dependent on A. When A fails, then $P(A') \text{ and } P(B \text{ given } A') = P(A')*P(B \text{ given } A') = 0.1*0 = 0$. (3) B1 and B2 are independent, then $P(B1 \text{ and } B2) = P(B1)*P(B2)$.

Scenarios				Scenario Likelihood	Outcome (End State)	Outcome Likelihood	
Initiating Event	A	B1	B2	$1*0.9*0.8*0.7 = 0.504$	Success	0.846	
			B2'	$1*0.9*0.8*0.3 = 0.216$	Success		
		B1'	B2	$1*0.9*0.2*0.7 = 0.126$	Success		
			B2'	$1*0.9*0.2*0.3 = 0.054$	Failure		
		A'					0.154
	Sum of Likelihoods				1.000		1.000

- Method 2 - Solve via Probability Formulas:

P(System X success): From English $P(S) = P[A \text{ and } (B1 \text{ or } B2)]$ to Mathematics $P(A)*[1-(1-P(B1))*(1-P(B2))] = (0.9)*[1-(1-0.8)*(1-0.7)] = (0.9)*[1-(0.2)*(0.3)] = (0.9)*[0.94] = 0.846$.

P(System X failure) = $1-P(\text{System X success}) = 1-0.846 = 0.154$. Another method is $P[(A \text{ and } B1' \text{ and } B2') \text{ or } (A')] = (0.9*0.2*0.3) + (.1) = 0.054 + 0.1 = 0.154$.

- Extra - Quantitative Risk (as an alternative to a risk matrix):

System X expected risk = $P(\text{System X failure})*(\text{System X failure consequence}) = (0.154)*\text{Consequence}$.

Data: Failure, Hazards, and Telling the Story

- ◆ For a photocopier, **which events are a failure?** It depends on the mission and your “[model of the world](#).”

- | | | | |
|---------------------|---------------------|---------------------|------------------------|
| • Is old but works | • Has cracked glass | • Is out of toner | • Is in use by others |
| • Does not do color | • Will not power up | • Is being repaired | • Is not permitted for |

- ◆ “**To understand system assurance**, one has to understand the definition of a failure and hazard. If a system does not meet the reasonable expectation of the user, then it has failed, even though it meets the specifications. When failures result in hazards, accidents can occur.”

Source: Assurance Technologies Principles and Practices, 2nd ed., Raheja & Allocco 2006, p. 5

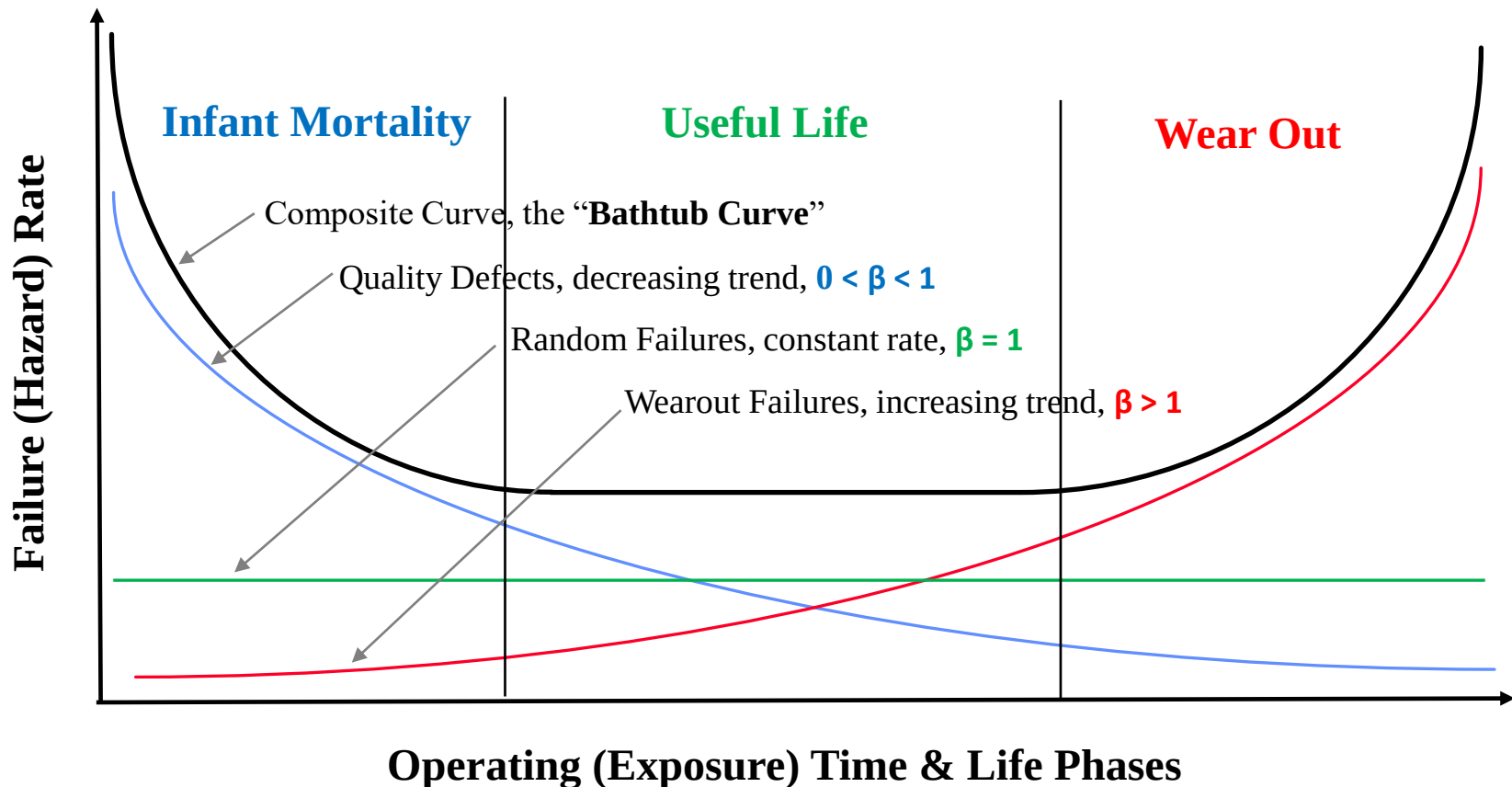
- ◆ **Vocabulary**: [Electropedia \(IEV Online\)](#), a resource from [IEC](#), prepares and publishes international standards.

Term	IEV ref	Definition
• Failure of an item	192-03-01	Loss of ability to perform as required. Also, see 192-03-03
• Hazard	903-01-02	Potential source of harm ... qualified with origin (e.g., fire hazard).

- ◆ **Related Concepts**: **Item** = hardware, software, orgware (humans), interfaces, or combination. **Safety** = freedom from accident and loss. **Risk** (in failure space) = potential loss. **RMA** = defined on slide [4](#).
- ◆ **Data Storyline**: Failure Event (item’s what, when, & where) → Failure Mode (observed what & how much) → Failure Mechanism (why did it fail; causes) → Failure Reoccurrence Control (how to prevent, mitigate, respond to).

Tip: For RMA data, at least collect **operational data**: Operating behaviors and outcomes, inferred by the design model, non-physical characteristics, and uses **run time and success counts** (example: [slide 5](#)). **Technical data**: Functional capability, contained in the design model, **physical characteristics**, and uses various units of measure.

Definitions: Bathtub Curve and Failure Rate Types

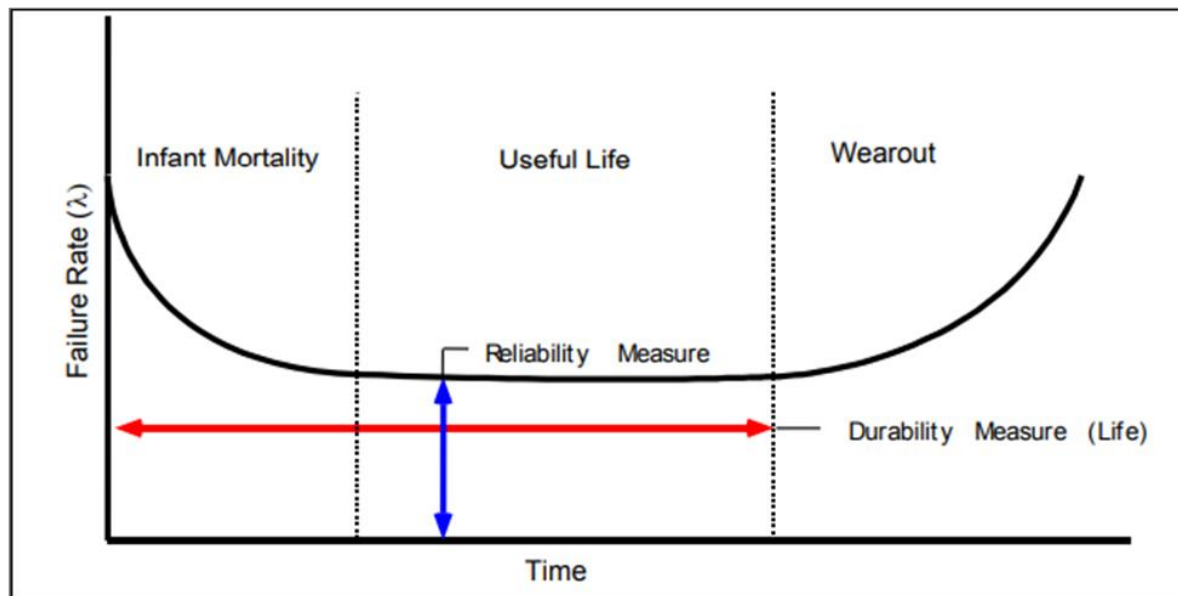


The **Bathtub Curve**, a **notional concept**, combines three types of failure (hazard) rate functions, $\lambda(t)$, over an item's life. Included is the Weibull probability distribution's shape parameter (β) for each failure trend type.

Definition: Durability

- ◆ “**Durability** is usually defined by the length of a failure free or maintenance free operational period. The basic assumption is that all failures are caused by applied mechanical/thermal stresses, and that there are no failures before the end of the failure free period (useful life) is reached.” **Failure rate** (λ) is the height during durability.

Source: System Reliability Toolkit-V, 2015, p. 575



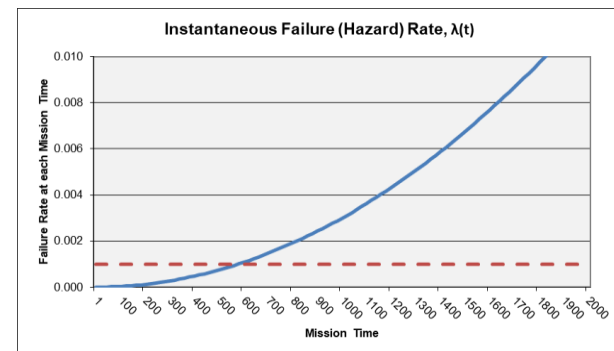
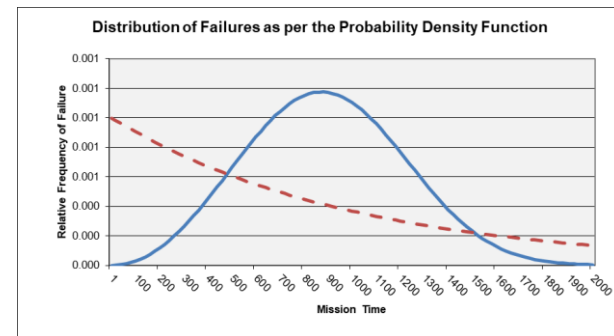
Over the item's entire life, the “**Reliability Measure**” is a function or functions of time, t . Denoted $\lambda(t)$ or $h(t)$.

When the value of $\lambda(t)$ is a constant (c), the expression is commonly written as $\lambda = c$.

For example, $\lambda = 0.001$ f/h means 1 failure every 1000 hours, a rate that remains the same over the item's lifetime or specific to the “Useful Life” phase.

Exponential Distribution: What does it mean?

- ◆ For large n , $e \approx (1+1/n)^n$ or $\sum_0^\infty 1/n! \approx 2.718282$
- ◆ With money, the growth factor for *continuously compounded* interest at i per period for t periods is: $e^{(i*t)}$. At $i = 6.93\%/yr$ and $t = 10$ yrs, factor = 2.
- ◆ As a probability distribution, when λ is the failure rate (failures/hr) and t is mission duration (hrs), the probability of mission success is: $ps = e^{(-\lambda*t)}$.
- ◆ Observe: Interest i and failure rate λ are constant rates over time. Use Weibull when λ is not constant.
- ◆ From a failure viewpoint, **a constant failure rate over time means**: (1) Previous exposure time (age) and stresses are not remembered, (2) Failures are due to random stresses (loads) beyond the intent (strength or capability) of the item's design, and (3) Preventive maintenance provides no benefit to reliability.
- ◆ From a probability viewpoint, **when the exponential distribution is the math model** and mission time is the mean time between failure (**MTBF** = $\theta = 1/\lambda$), then: $ps = e^{-1} \approx 0.367879$ and $pf = 0.632121$.
- ◆ Legend: **Red** is the exponential distribution; **blue** is the Weibull distribution. Both use a scale parameter (θ) = 1000 time units. With the exponential distribution, the scale parameter is the MTBF. The Weibull below used a shape parameter (β) = 3 which means: **When β is greater than one**, failure rate is increasing (not constant) over time.



Data: Types and Math Models

- ◆ For the item under study, **data includes**: (1) Both failures and non-failures (censored data) and (2) **Applicable areas**: hardware, software, orgware (humans), and interfaces.
- ◆ Data types (more on [slide 20](#)) and common (not the only) math models:

1. Time-based (clock) data

- **Continuous** (e.g., jet engine run hours)
- Lifetime math model: **Weibull**
- Repair time math model: **Lognormal**

2. Event-based (demand) data

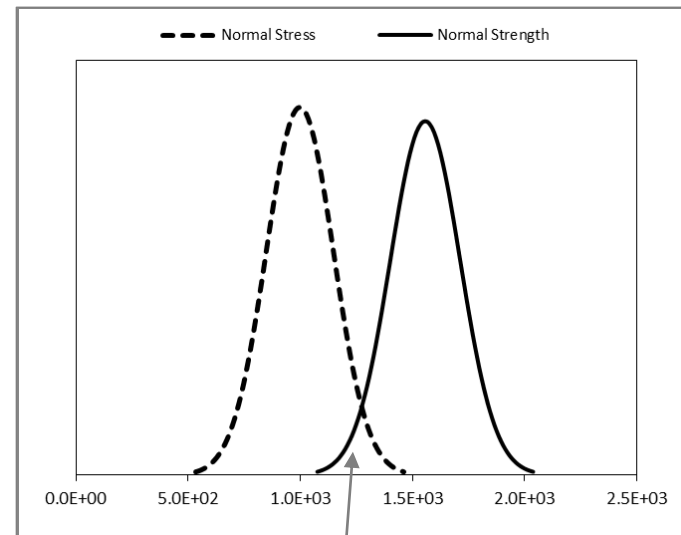
- **Discrete** (e.g., landing gear actuations)
- Math models: **Binomial** and **Poisson**

3. Stress (load) and strength (capacity) data

- Two **Normal** distributions are used →
- **Note**: A safety factor does not characterize the uncertainty in the item's stress and strength.

4. Combination

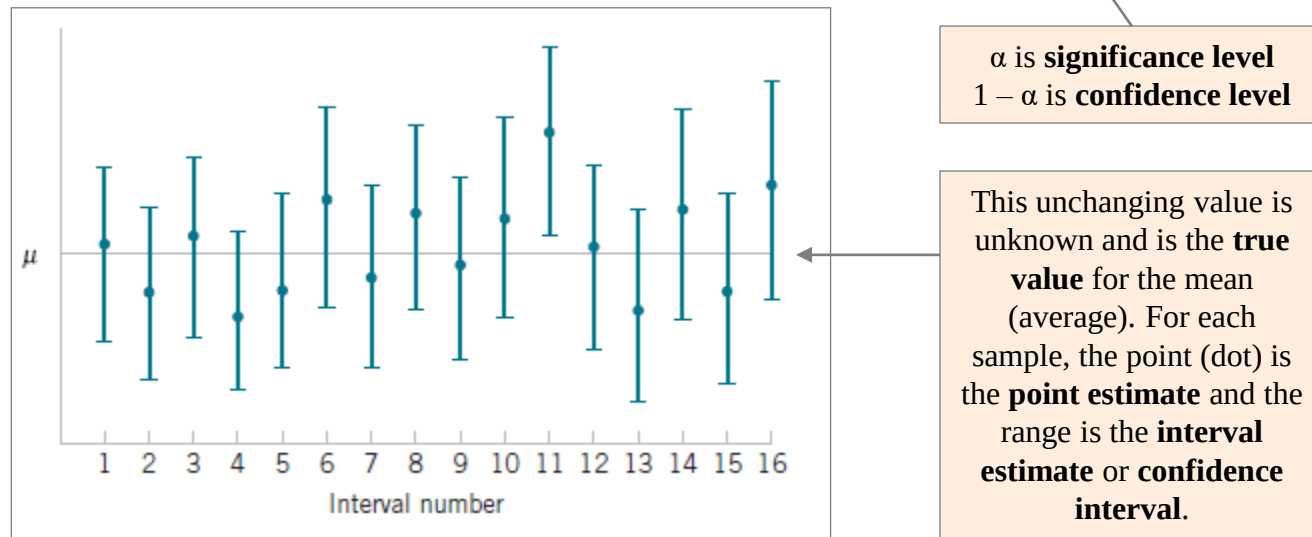
- Time-to-failure data at different stress levels
- Common math model: **Covariate Weibull**



This area corresponds to the **probability of failure** due to variation (uncertainty) in stress and strength.

Confidence Interval: What does it mean?

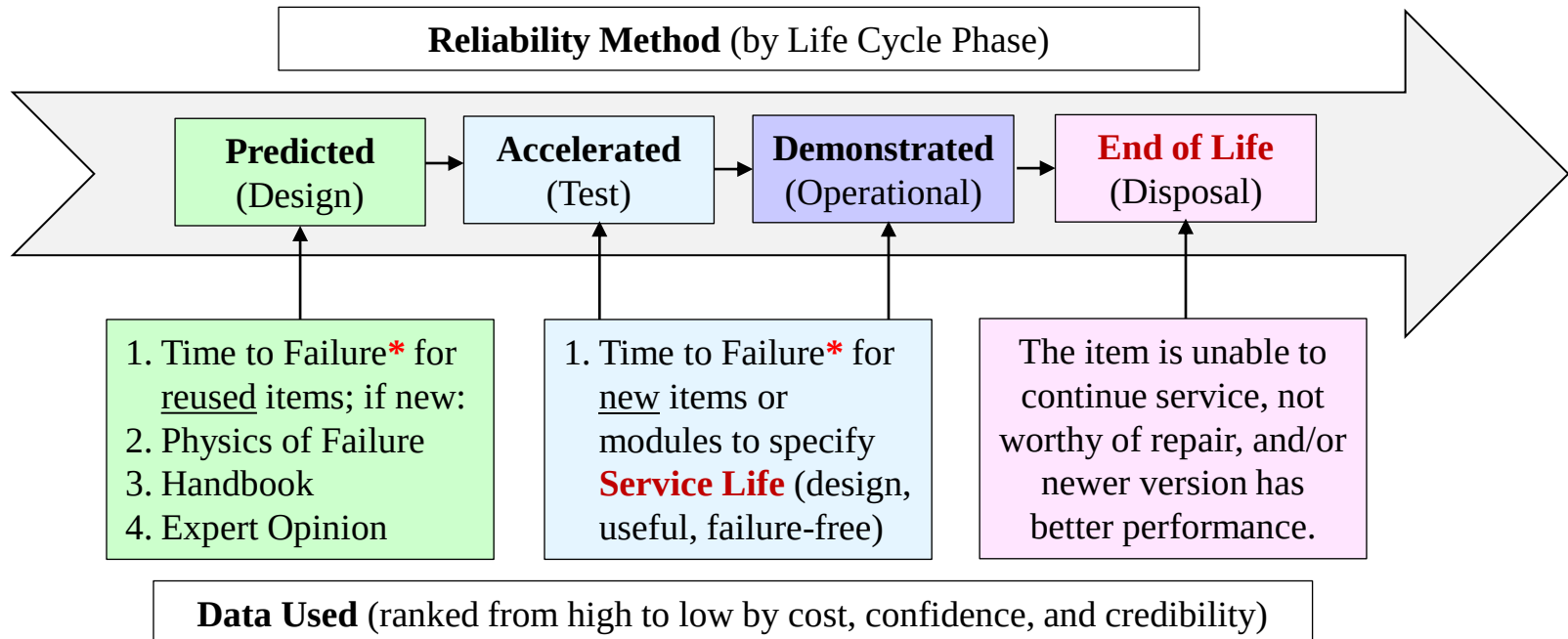
- ◆ “... in practice, we obtain only one random sample and calculate one confidence interval. Because this interval either will or will not contain the true value of μ (mean), it is not reasonable to attach a probability level to this specific event. The appropriate statement is that the observed interval [lower, upper] brackets the true value of μ with confidence $100(1 - \alpha)$. This statement has a frequency interpretation; that is, *we do not know whether the statement is true for this specific sample, but the method used to obtain the interval [lower, upper] yields correct statements $100(1 - \alpha)\%$ of the time.*”



Source: Applied Statistics and Probability for Engineers, 7th ed, Montgomery & Runger. 2018, p. 174

Quantitative Reliability: Method vs. Data Used

(Other words, quality of the forecast vs. quality of the data used)

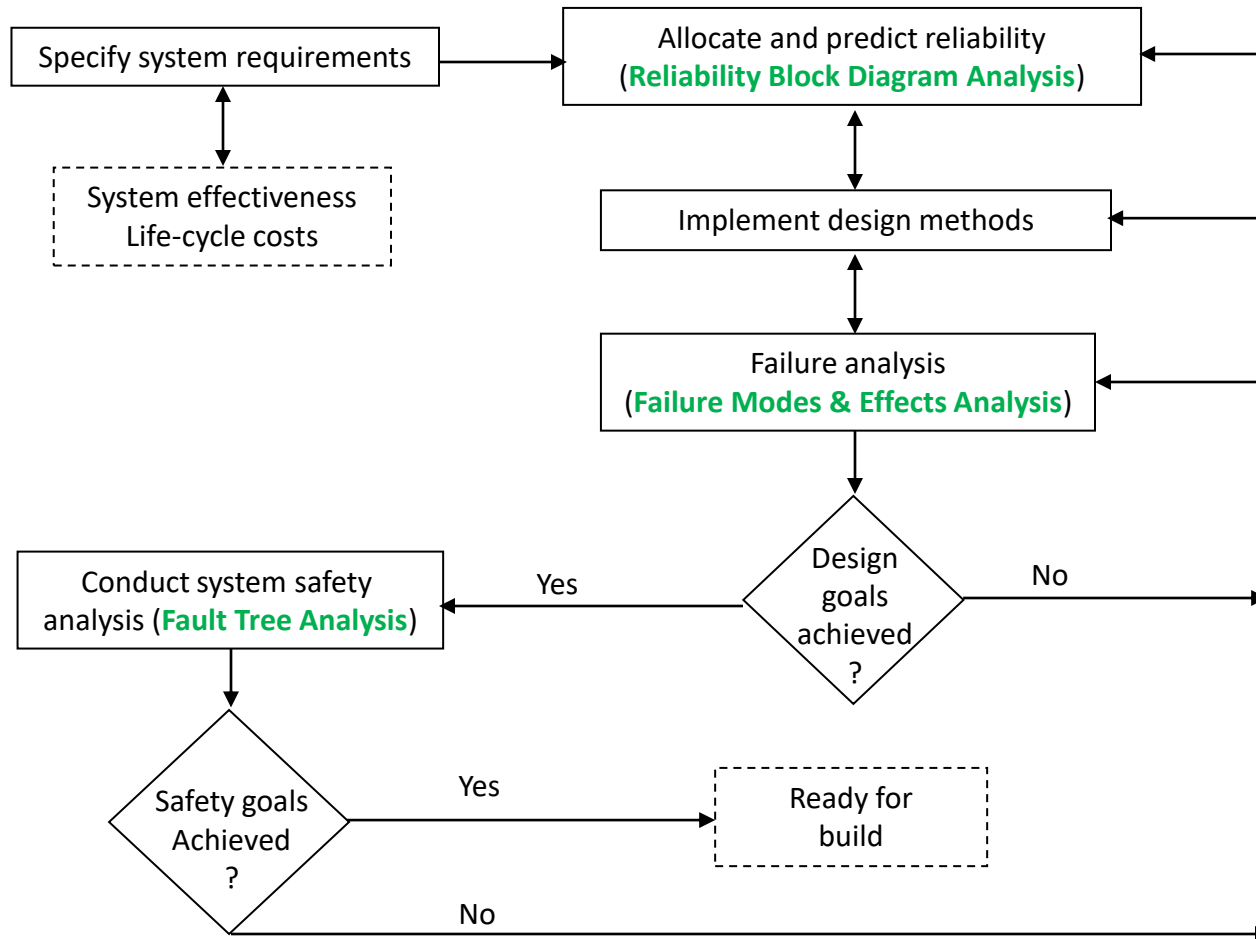


* **Recorded data** for each item is the time-to-failure (TTF) or the run (censored) time if the item that has not failed.

Reliability Verification Testing (Data Type #1): Demonstrates with minimum test time (provides **operational data**, slides 5, 6, and 14) and sample size that the item meets the reliability target (allocated from the mission success goal).

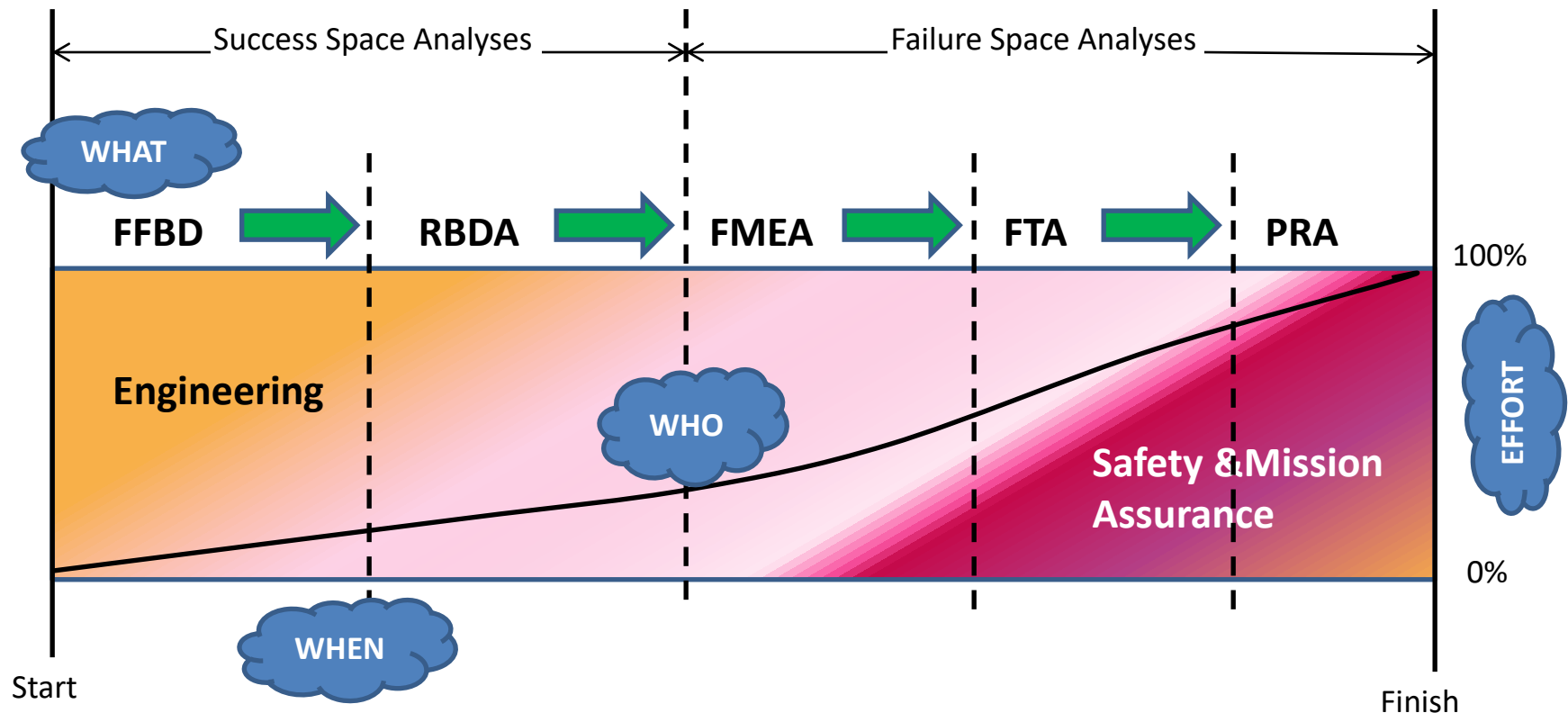
Analytical Reliability Verification (Data Types #2, #3, and #4): Is used when testing is too expensive (\$) and time consuming. Instead, reliability is verified when there are adequate mathematical models that relate the item's life to stresses, design parameters, and manufacturing variables. (ref. Life Cycle Reliability Engineering, Yang, 2007, p. 54)

Risk (Safety & Reliability) Analyses in the Design Phase



Source: An Introduction to Reliability and Maintainability Engineering, Charles Ebeling, 2005, Figure 8.1.

An Idealized Sequence for Engineering Assurance Analyses



Analytical Products:

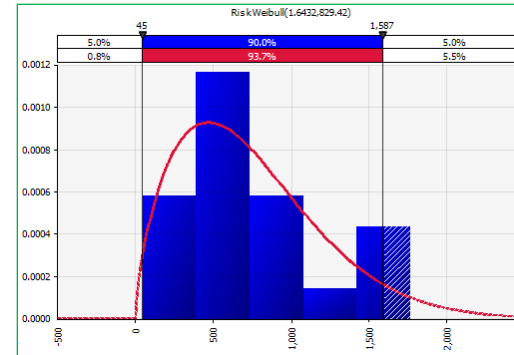
FFBD = Functional Flow Block Diagram
RBDA = Reliability Block Diagram Analysis
FMEA = Failure Modes & Effects Analysis
FTA = Fault Tree Analysis
PRA = Probabilistic Risk Assessment

Theme:

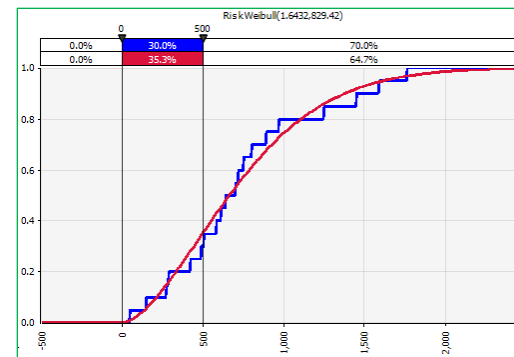
This work sequence (WHEN) builds and uses analytical products (WHAT) in an optimum manner—especially during the Design Phase. The appropriate mix of experts (WHO and EFFORT) make and deliver the right analytical product at the right time. In addition to serving the intended purpose at the desired time, each analytical product serves as an input that expands the technical fidelity of analytical products that follow.

Find pf: Given time-to-failure (TTF) data (identical tests)

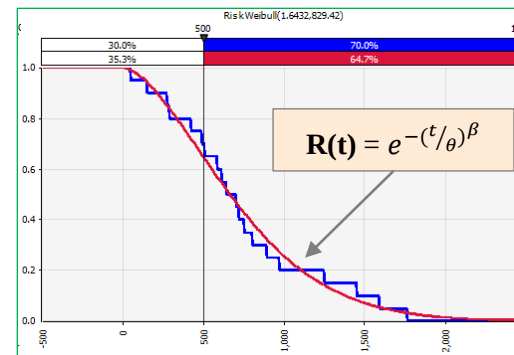
- ◆ **Question:** As a point estimate (not interval estimate), **what is the probability** item X when new will perform its intended function under stated conditions for a mission time (t_m) equal to 500 hours? Denoted as: $p_s = R(t_m) = \Pr[T > t_m] = ?$
- ◆ **Given:** The **time-to-failure** (TTF) data for **20 identical items on test under the stated conditions**: 716, 1451, 425, 1763, 1249, 283, 801, 752, 149, 585, 697, 968, 611, 510, 1587, 489, 641, 274, 893, and 45. **Note:** The mean or **average is 744.45 hours**. **Tip:** When applicable, include the time for items that have not failed (censored data).
- ◆ **Work Process:** data → histogram → postulate model and select estimation method → probability density function, $f(t)$ → cumulative distribution function, $F(t) = \int f(t) dt$ for $[0, \infty] \rightarrow 1 - F(t) = R(t)$, the Reliability model → Use $R(t)$ where $t = t_m \rightarrow R(t_m) = \text{probability of success } (p_s)$.
- ◆ **Answer:** Using the two-parameter Weibull probability distribution as the math model and the Maximum Likelihood Estimation (MLE) method, the model's shape parameter (β) ≈ 1.6432 and scale parameter (θ) ≈ 829.42 hours. Thus, **for this math model** based on actual data, when mission time = 500 hours, reliability ≈ 0.6470 .
- ◆ **Report:** Under the same build and operating conditions, item X has a 64.7% chance in performing its intended function for more than 500 hours; **35.3% chance in not performing its intended function** for 500 hours or less. Denoted as: $p_F = F(t_m) = \Pr[T \leq t_m] \approx 35.3\%$.



Histogram and the selected Probability Density Function (PDF), $f(t)$



Cumulative Distribution Function (CDF), $F(t)$



Reliability Function, $R(t) = 1 - F(t)$

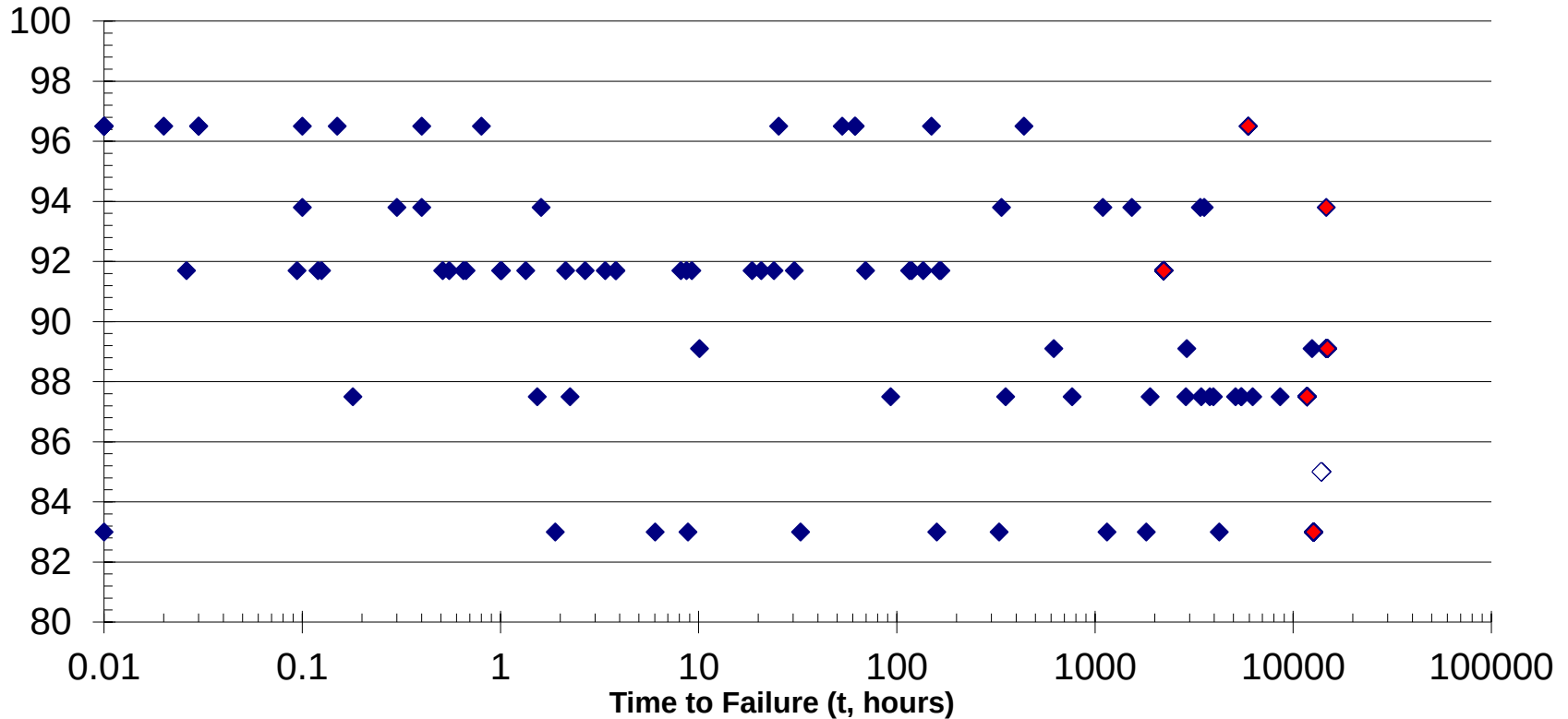
Find pf: Given time-to-failure (TTF) data (not identical tests)

- ◆ **Testing at different stress levels** has the purpose to reduce the length of the test period by accelerating failures in highly reliable items.
 - This type of testing is commonly called **Accelerated Life Testing (ALT)**.
 - The data from an ALT comes from a test plan.
 - **For example**: At least three levels of stress and six failures at each stress level.
 - **For example**: The **next slide** provides actual test data from a planned ALT.
- ◆ **Tools for Accelerated Life Testing**:
 - Author's **Excel tool**.
 - Based on the Weibull probability distribution.
 - All items on test must fail.
 - KSC's [PTC Windchill Reliability and Risk \(WRR\) Tool Suite](#) [NASA **internal**].
 - WRR is on the KSC network and has an ALT module.
 - For details on WRR's ALT module, go to Section 2-1 in the [PTC WRR Reference Guide](#) [NASA **internal**].

Question: Based on this test data, what is the probability of success when the stress ratio is 80% and mission time is 150 days?

Percentage of
Operating to Burst
Stress (s)

Legend: Blue = Time of failure; Red = No failure since test stopped



Find pf: Given only runtime or test count (no failures)

- ◆ The author made an Excel tool for **zero failures**. This tool finds the probability of success (or reliability) and the probability of failure when the recorded data has **no failures** either **by chance** or **by design**.
- ◆ The recorded data can be runtime (exposure, duration of the load) or the number of trials (tests, demands).
- ◆ For example, this tool answers questions such as:
 1. **Can testing stop** after receiving *no failures* in 360 pass-fail tests? The risk goal is 4/10,000.
 2. **How many units must be tested** for 300 hours to demonstrate 0.99 reliability with 80% confidence for a 200-hour mission? Assume:
 - a. The item's failure rate is a Weibull with a shape parameter β from 2 to 3.5.
 - b. The most efficient test design is desired – that is, *no failures*.
 3. **Which zero-failure-test-design** [in a provided table] predicts that an item with a 5-minute mission time has a 0.99 reliability with a 95% statistical confidence?

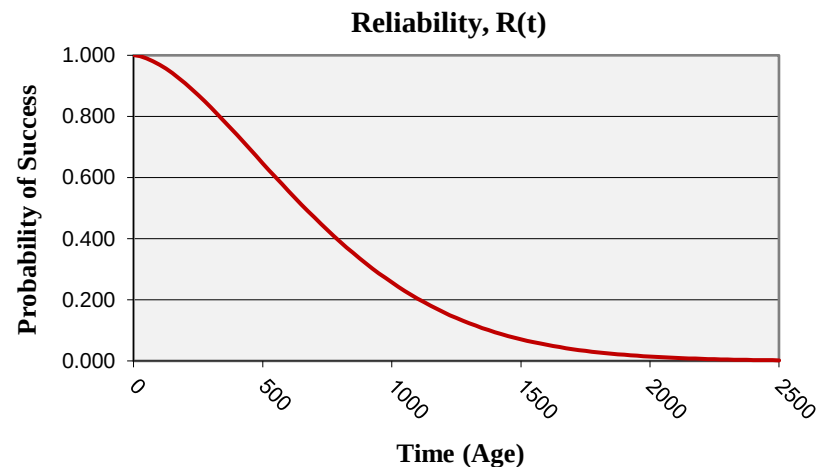
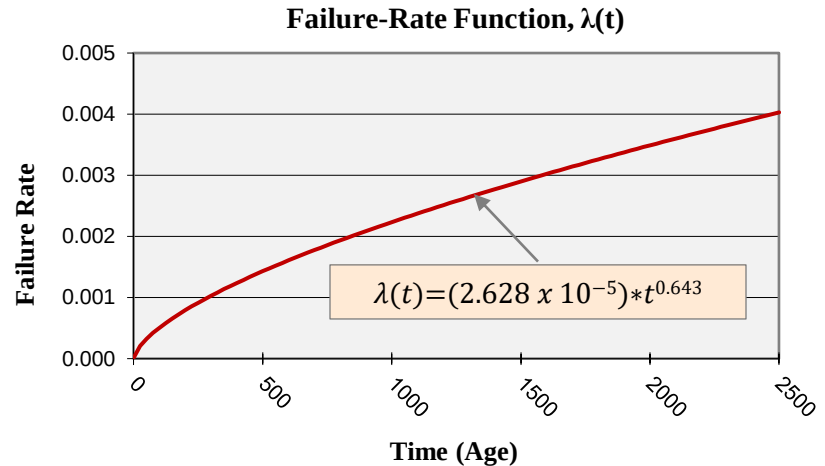
Find pf: Given static stress and strength (with variation)

- ◆ The author has an **Excel tool** for **Stress-Strength Interference** that finds the probability of success and the probability of failure when the input data provides the:
 - Mean (average) and standard deviation (variation) of the stress (load) and the strength (capacity).
 - Probability distribution (typically, normal for physical measurements) for stress and strength.
- ◆ **For example:**
 - Given: A pressure tank rated at 525 psi was planned to be used at 0.33 stress ratio. Later, the requirement changed to 0.45 stress ratio. Instead of standard deviation, the Coefficient of Variations (std dev/mean) were provided by expert opinion, namely, 5% for stress and 15% for strength.
 - Question: As a factor, how much does risk likelihood (pf) increase when the stress is increased?
 - Answer:

IF					THEN			
Case	Assumptions				Probability of Failure	Safety Factor	Safety Index, β	Stress Ratio
A	Stress, Distribution	Normal						
	Stress, Mean		236.250					
	Stress, Variability			11.813				
	Strength, Distribution	Normal						
	Strength, Mean		525.000					
	Strength, Variability			78.750				
	Measures →				1.44E-04	2.22	3.63	0.45
B	Stress, Distribution	Normal						
	Stress, Mean		175.000					
	Stress, Variability			8.750				
	Strength, Distribution	Normal						
	Strength, Mean		525.000					
	Strength, Variability			78.750				
	Measures →				5.00E-06	3.00	4.42	0.33
Comparison Factor (Ratio)			Case A/Case B		28.78	0.74	0.82	1.35
			Case B/Case A		0.03	1.35	1.22	0.74

Find pf: Given failure-rate equation, $\lambda(t)$ (model, no data)

- ◆ Question: As a point estimate (not interval estimate), what is the probability item X when new will perform its intended function under stated conditions for a mission time (t_m) equal to 500 hours?
- ◆ Given: $\lambda(t) = (2.628 \times 10^{-5}) * t^{0.643}$, failure-rate function (or hazard function) for item X from the lab, a handbook, or journal paper. Note: Confirm before using $\lambda(t)$ as a model for degradation over time.
- ◆ Work Process: Use $R(t_1, t_2) = \text{Exp} \left[- \int_{t_1}^{t_2} \lambda(x) dx \right]$ where t_1 = age and t_2 = age + mission time = $t_1 + t_m$. Since item X is new when used, then $t_1 = 0$ which makes $t_2 = 0 + 500$ hours.
- ◆ Answer: After integrating $\lambda(t)$ using the limits $[0, 500]$, $R(0, 500) = e^{-0.4353} \approx 0.6471$.
- ◆ Report: Under the same conditions (e.g., build and operate), a new item X has a 64.7% chance in performing its intended function for more than 500 hours; 35.3% chance in not performing its mission for 500 hours or less.



Find pf: Other analytical approaches (design parameters & parts)

◆ *Physics of Failure (PoF) or Reliability Science. Examples on next slide.

- [NASA Physics of Failure Handbook](#)
- [Overview of Reliability Science \(Tutorial\)](#)
- [Mechanical parts and assemblies \(US Navy's Mechrel\)](#)
- [Electronic parts and assemblies \(Univ of Maryland's SARA\)](#)

***PoF** is how it should perform under specified conditions. **Statistical modeling** is how it did perform.

◆ Failure-rate handbooks

- For example: [Quanterion Databooks](#), [FIDES](#), and [ANSI/VITA](#) (e.g., 51.1 & 51.2)

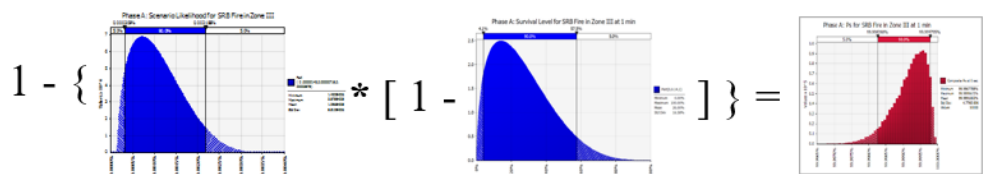
◆ Expert opinion

- **This example** characterizes uncertainty in the expert's view (input) on the likelihood of the event occurring (P_E) and the likelihood of survival given the event occurred ($P_{S|E}$) and the likelihood of survival (P_S).

If the Excel formula is:

$$1 - \{ P_E * [1 - P_{S|E}] \} = P_S$$

Then the @RISK tool with expert opinion makes:



Examples: Physics of Failure (PoF)

From Design for Reliability, Crowe & Feinberg, 2001

Temperature and Humidity

Related Failures - Peck Model

$$A_T = \exp\left\{\frac{E_a}{K_B} \left[\frac{1}{T_{Use}} - \frac{1}{T_{Stress}} \right]\right\}$$

$$A_H = \left(\frac{R_{Stress}}{R_{Use}}\right)^m$$

$$A_{TH} = A_T A_H$$

$$\ln(t_f) = C + \frac{E_a}{K_B T} - m \ln(R)$$

Notation

A_H = humidity acceleration factor
 A_T = temperature acceleration factor
 A_{TH} = temperature-humidity acceleration factor
 R_{Stress} = relative humidity of test
 R_{Use} = nominal use relative humidity
 T_{Stress} = test temperature
 T_{Use} = nominal use temperature
 m = humidity constant
 E_a = activation energy
 t_f = time to fail
 C = constant

Temperature Related Failures

- Arrhenius Model

$$A_T = \exp\left\{\frac{E_a}{K_B} \left[\frac{1}{T_{Use}} - \frac{1}{T_{Stress}} \right]\right\}$$

$$\ln(t_f) = C + \frac{E_a}{K_B T}$$

Notation

A_T = temperature acceleration factor
 T_{Stress} = test temperature (°K)
 T_{Use} = use temperature (°K)
 E_a = activation energy
 K_B = 8.6173×10^{-5} eV/°K
 (Boltzmann's constant)
 t_f = time to failure
 C = constant

Vibration Related Failures

- MIL-STD 810E

$$A_V = \frac{T_{Use}}{T_{Stress}} = \left(\frac{W_{Stress}}{W_{Use}}\right)^{M_b}$$

$$\left(\frac{W_{Stress}}{W_{Use}}\right)^{M_b} = \left(\frac{G_{f,Stress}}{G_{f,Use}}\right)^{2M_b}$$

$$\ln(t_f) = C - M_b \ln(W)$$

Notation

A_V = vibration acceleration factor
 T_{Stress} = vibration duration
 T_{Use} = vibration duration (nominal)
 W = random vibration input PSD across the resonance bandwidth (G²/Hz)
 W_{Stress} is the PSD test stress and
 W_{Use} nominal use PSD
 G_f = resonant G sinusoid vibration level
 M_b = b/2 where b is the fatigue parameter
 t_f = time to failure
 C = constant

Temperature Cyclic Related Failures

- Coffin Manson Model

$$A_{TC} = \frac{N_{Use}}{N_{Stress}} = \left(\frac{\Delta T_{Stress}}{\Delta T_{Use}}\right)^K$$

$$\ln(N_f) = C - K \ln(\Delta T)$$

Notation

A_{TC} = temperature cycle acceleration factor
 N_{Stress} = number of cycles tested
 N_{Use} = equivalent number of field cycles
 ΔT_{Stress} = temperature cycle test range
 ΔT_{Use} = nominal daily temperature change in the field
 K = temperature cycle exponent
 N_f = number of cycles to failure
 C = constant

Wearout (Fatigue) Failures via

Uniform Cyclic Load - S-N Curve

$$N = cS^{-m}$$

Notation

N = median number of cycles to failure
 S = magnitude of the cyclic stress
 c and m = constants determined experimentally

Data Analysis

The log-log plot of the stress S and the number of cycles N to failure is called the **S-N curve**.

The S-N curve indicates the cycles to failure at any cyclic stress value between the ultimate stress and the fatigue limit (endurance limit).

Log S vs. Log N data tend to fall along a straight line. The values for c and m can be obtained from a least squares analysis.

Electromigration Failures -

Black equation

$$A_I = \left(\frac{J_{Stress}}{J_{Use}}\right)^n \exp\left\{\frac{E_a}{K_B} \left[\frac{1}{T_{Use}} - \frac{1}{T_{Stress}} \right]\right\}$$

$$\ln(t_f) = C + \frac{E_a}{K_B T} - n \ln(J)$$

Notation

A_I = electromigration acceleration factor
 T_{Stress} = test temperature (°K)
 T_{Use} = use temperature (°K)
 E_a = activation energy
 K_B = 8.6173×10^{-5} eV/°K
 (Boltzmann's constant)
 J = current density
 n = current density exponent
 t_f = time to failure
 C = constant

Find pf: Given upper limit and test data parameters

◆ Customer's Request:

- What is the likelihood of failure (pf)? The customer defines success when the duration of a specified task is less than 60 minutes.
- What can the customer do to reduce the risk at least with the likelihood of failure?

◆ Given:

- The lab designed a test that measured the duration to complete the specified task.
- The lab conducted 20 tests.
- The mean (average) of the 20 durations is 55 minutes.
- The standard deviation (a measure of spread or dispersion of the recorded data) is 3 minutes.
- Use a 95% two-sided statistical confidence to address parameter (statistical) uncertainty.

◆ Method:

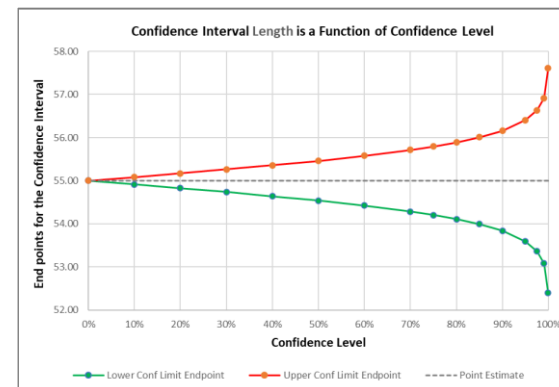
- Since the sample size is less than 30, then use the “t Distribution” to find the confidence interval.
- Find and compare the average duration (mean, expected value) for the both the point estimate and upper limit of the confidence interval.

◆ Findings:

- **Baseline:** (1) Probability of failure (pf) is 0.0478 (4.8%) when average time uses the point estimate. (2) Probability of failure (pf) is **0.1153** (11.5%), a factor of 2.41 when average time uses the upper limit of the 95% confidence interval.
- **Mitigation scenarios upper limit pf:**

Variable	Change	Upper pf
Sample Size	↑50%	0.0980
Confidence	↓15.8%	0.0854
Variation	↓10%	0.0832
Upper Limit	↑1%	0.0810

– Confidence: Interval Length vs. Level:



Need to **accept or reject** a claim: Then use statistics

Source: Applied Statistics and Probability for Engineers, 7th ed, Montgomery & Runger. 2018, p. 211 and [Anastasiia Kim's Lecture](#)

◆ Given:

- A crew escape system is powered by a solid propellant. The burning rate of this propellant is an important product characteristic and follows a Normal distribution.
- Specifications require that the mean burning rate must be 50 centimeters per second (cm/sec).

◆ Method - Procedure for Hypothesis Tests:

1. **Parameter of interest:** From the problem context, identify the parameter of interest.
2. **Null hypothesis, H_0 :** State the null hypothesis, H_0 .
3. **Alternative hypothesis, H_1 :** Specify an appropriate alternative hypothesis, H_1 .
4. **Test statistic:** Determine an appropriate test statistic.
5. **Reject H_0 if:** State the rejection criteria for the null hypothesis.
6. **Computations:** Compute any necessary sample quantities, substitute these into the equation for the test statistic, and compute that value.
7. **Draw conclusions:** Decide whether or not H_0 should be rejected and report that in the problem context.

◆ Findings:

- The standard deviation of burning rate is $\sigma = 2$ cm/sec. The experimenter specified a type I error probability or significance level of $\alpha = 0.05$ and selects a random sample of $n = 25$ and obtains a sample average burning rate of $\bar{x} = 51.3$ cm/sec.

◆ Solution - Using the 7-Step Method:

1. Parameter of interest is μ , the mean burning rate.
2. $H_0: \mu = 50$ cm/sec. 3. $H_1: \mu \neq 50$ cm/sec.
4. Since Normal with known σ : $z_0 = (\bar{x} - \mu_0) / (\sigma / \sqrt{n})$
5. Reject H_0 if the P-value < 0.05 . To use a fixed significance level test, the critical region would be $z_{0.025} = 1.96$ and $-z_{0.025} = -1.96$.
6. Because $\bar{x} = 51.3$ and $\sigma = 2$, $z_0 = 3.25$.
7. Because the P-value $= 2[1 - \Phi(3.25)] = 0.0012$ we reject $H_0: \mu = 50$ at the 0.05 level of significance.

◆ Interpretation:

- The mean burning rate differs from 50 cm/sec based on a sample of 25 measurements. There is strong evidence the mean burning rate exceeds 50 cm/sec. Thus, **the claim $\mu = 50$ cm/sec is rejected.**

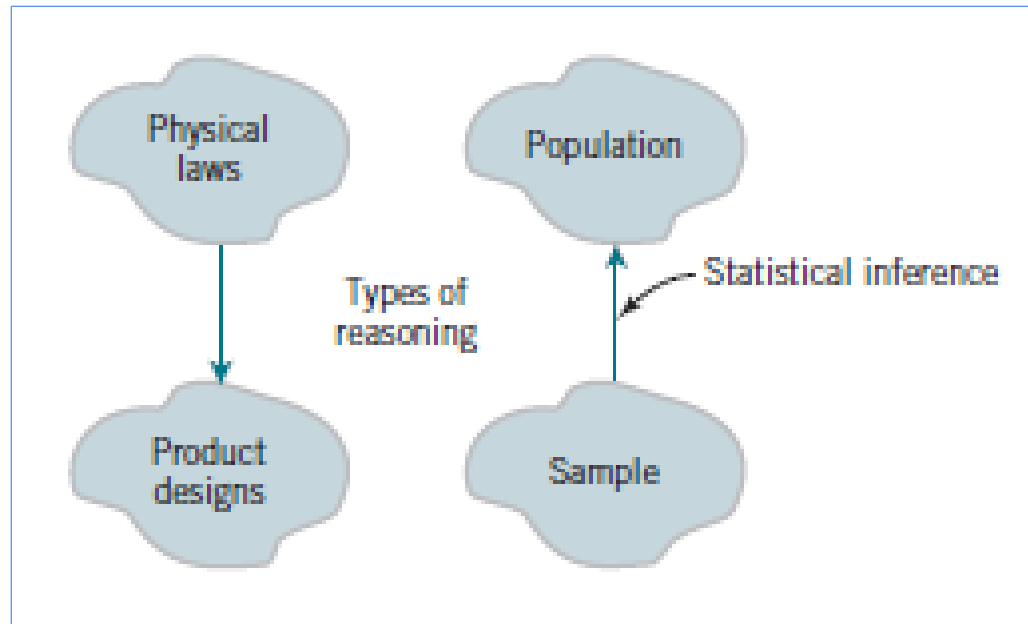
Acknowledgments

- ◆ My thanks to Peter Taddie who encouraged me to write this presentation. “Pete” is the Chief of the Engineering Directorate’s Materials & Engineering Processes Branch at the NASA Kennedy Space Center.

Summary

To get started, remember ...

Statistical inference is one type of reasoning.



Decisions often need to be based on measurements from only a subset of objects selected in a sample. This process of reasoning from a sample of objects to conclusions for a population of objects is referred to as **statistical inference**. Probability models help quantify the risks involved in statistical inference.

Source: Applied Statistics and Probability for Engineers, 7th ed., Montgomery & Runger, 2018, pp. 5 & 15

To collect **operational data:** Organize the time-to-failure (TTF) and censored data to simplify the construction and audit of the probability distribution.

Unique Item (Serial Number)	Time to Failure (TTF) <u>or</u> Run Time with No Failure (hours)				Interpretation
	Record 1	Record 2	Record 3	Record 4	
1	1000 C				(1) "C" = Censored time , item's cumulative run time without failure at the time of the study. (2) Record = The item's age recorded either as a censored time or time to failure (TTF). One item still works or is able to work after 1000 hours of cumulative or continuous run time.
2	1000				One item failed after 1000 hours of cumulative or continuous run time. No corrective action (repair or replacement) was performed.
3	1000	200 C			One item failed after 1000 hours of cumulative or continuous run time. The failed item was repaired (not replaced) and still works or is able to work after 200 hours of use.
4	600	500	400	300 C	One item failed three times and was repaired (not replaced) three times. After the third repair, the item still works or is able to work after 300 hours of use.
5	600				Four items at one location with three replacements , or four locations with no corrective action at three locations.
6	500				<u>Note:</u> S/N 4 (a reparable item) and S/Ns 5, 6, 7, and 8 (four non-reparable items) have the same reliability when the corrective maintenance made to S/N 4 is as good-as-new. This assumes to restore the item to its original condition, replacing the item with new is more effective in regard to reliability than repairing the item. <u>Tip:</u> For repairable items, perform a Laplace Test before using a life model based on order statistics. The Laplace Test uses life data in the order as occurred and not reordered from small to large. If the test score <i>equals zero</i>, the trend for the times to failure (TTF) is constant (item returned to its original condition; proceed with a Weibull analysis). If the test score <i>is less than zero</i> (e.g., -1 or less), the TTF is decreasing (life is degrading). If the test score <i>is greater than zero</i> (e.g., +1 or more), the TTF is increasing (life is better than original design).
7	400				
8	300 C				