Binomial Distribution And The “k-out-of-n” Case

1. Requirement: You need k number of successes from n number of trials (attempts, paths, legs, or missions). Or do you need no less than k number of successes? The mathematics is different for each case.

2. Modeling: There are five measurement relationships between k and n. The table below illustrates the five relationships for $k = 2$ and $n = 4$. For your work, use the relationship appropriate for the decision to be made.

$<$	Less than k	●	●			
$\leq$	k or less	●	●	●		
$=$	Exactly k			●		
$\geq$	k or more			●	●	●
$>$	More than k				●	●
		0	1	k = 2	3	n = 4
		Trials (attempts, paths, legs, or missions)				

3. Properties: Use the binomial probability distribution function to model the five cases above if:

- The number of trials (n) is fixed and is not infinite.
- The likelihood, or probability, of success (p_s) is the same from trial to trial. Common cause failure is not addressed.
- The trials are independent of one another. Example: Two trials in the same sample space are said to be independent if the occurrence of one trial does not affect the occurrence of the other. Typically, the independence property is present with the binomial distribution since sampling is “with replacement” as opposed to “without replacement.”

NASA KSC, Tim Adams, 321-867-2267, tim.adams@nasa.gov

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4. Formula: The **binomial probability distribution function** for the equal to (=) case is:

   The probability of exactly k successes out of n trials $= [\, n! \,/\, k!(n - k)!] * p_s^{k} * (1 - p_s)^{n - k}$ where:

   - k is the number of successes (success must be defined since it can be failure to another party),
   - n is the number of trials (attempts, paths, legs, or missions),
   - p_s is the probability of success for each trial,
   - $*$ means multiply, and
   - $!$ is the mathematical factorial notation (e.g., 4! = 4 * 3 * 2 * 1 = 24; 0! = 1 by definition).

5. Formula: The **k-out-of-n system formula** is typically the greater than or equal to (≥) case of the binomial distribution and is described as follows:

   The probability of k or more successes out of n trials $= \sum_{i = k}^{n} [n! \,/\, i!(n - i)!] * p_s^{i} * (1 - p_s)^{n - i}$

   Comment: The formula in #5 yields the same result as the formula in #4 when the probability for each value of k is determined and summed.

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6. Examples using the k-out-of-n system formula:

IF			THEN	
k or more successes are required	Out of **n** legs (of redundancy)	With an assumed probability of success (**p$_s$**) per leg	System Reliability (**R$_s$**), the probability of k or more out of n	Comment
0	3	.9	1.000	For $k = 0$, $R_s = 1$ regardless the values of n and p_s.
1	“	“	0.999	For $k = 1$, $R_s = 1 - (1 - p_s)^n$, traditional parallel system reliability.
2	“	“	0.972	For $1 < k < n$, traditional k-out-of-n system; use formula in #5.
3	“	“	0.729	For $k = n$, $R_s = (p_s)^n$, traditional series system reliability.
6	8	.8	0.797	A military planner has 8 helicopters to perform a rescue mission; six are required. Based on past experience, helicopter reliability is believed to be 0.80 for the duration of the mission. The helicopters work or fail independently of each other. What is the probability of a successful rescue? Was common cause failure considered? (Answer: No.)
100	100	399/400	0.779	A vehicle designer has been assigned a probability of failure goal of 1/400 per mission. What is the program or lifetime reliability if the vehicle is to be used 100 times, is expected to perform successfully 100 times, and the vehicle is returned to specs prior to each mission?
1	2	0.5255	0.774850	Each gyro must have at least 1 of its 2 hall effect sensors working. Sensor reliability is 0.5255 based on thermal cycling tests. All other gyro components are assumed to have a reliability of 1.000. What is the reliability of a gyro?
2	4	0.774850	0.962	A spacecraft has 4 gyros and at least 2 of the 4 must work. What is the likelihood the gyro system will not work for the mission? (Ans: About 4%.) Note: This is a nested application of the k-out-of-n system formula.