Formulas: Inherent Availability and Reliability with Constant Failure and Repair Rates[1]

	Point (Instantaneous) Availability at time t	**Average (Interval, Mission) Availability** during the time period from t_1 to t_2	**Limiting (Steady-State, Asymptotic) Availability** as time becomes large
Inherent Availability[2]	$A(t)_{sys} = \left[\frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t}\right]^N$	$A(t_1,t_2)_{sys} = \left[\frac{\mu}{\lambda+\mu} + \frac{\lambda}{(\lambda+\mu)^2(t_2-t_1)}[e^{-(\lambda+\mu)t_1} - e^{-(\lambda+\mu)t_2}]\right]^N$ This is point availability averaged over the time interval.	$A_{inherent-sys} = \left[\frac{\mu}{\lambda+\mu}\right]^N$ $= \left[\frac{MTBF}{MTBF+MTTR}\right]^N$
Reliability[3]	Reliability based on time pertains to a continuous interval of time and not at a point in time as with point availability. For any interval of time, time-based reliability is conditional probability. Conditional probability[3] is not the average of probabilities as with average (interval) availability. The form $R(t)_{sys} = \left[e^{-\lambda t}\right]^N = e^{-N\lambda t} = R(t_2 \mid t_1)$ is conditional reliability where $t_1 = 0$ and $t_2 = t$. When $t_1 \neq 0$ and $t_1 \leq t_2$, then $R(t_2 \mid t_1)_{sys} = \left[e^{-(\lambda)(\Delta t)}\right]^N = e^{-N\lambda\Delta t}$ where $\Delta t = t_2 - t_1$.		$R(t_2 \mid t_1) \to 0$ as $t_2 \to \infty$
Probability of r or less events[4]	Dropping the summation provides the probability of observing exactly r.	As a process, $P = \sum_{n=0}^{r} \frac{e^{-N\lambda t}(N\lambda t)^n}{n!}$ where t is from 0 to t_2.	$P \to 0$ as $t_2 \to \infty$

Notes:

1 - Nomenclature: **N** = number of elements in a series configuration; **MTBF** = element mean time between failure; **MTTR** = element mean time to repair; **λ** = 1/MTBF; **μ** = 1/MTTR; and **t** = mission time. λ, μ, and t have the same unit of time (e.g., hours). For design purposes, MTBF is a lower-bound parameter and MTTR is an upper-bound parameter. Note: λ and μ are constant rates over time. Thus, the exponential probability distribution is used to model the failure and repair distributions. From a reliability viewpoint, when the **assumption of constant failure rate** (CFR) is used along with the exponential distribution (the only continuous distribution that models CFR), the time to failure of an item is not dependent on how long the item has been operating. Thus, this lack of memory about the item's age is suitable when failures are primarily caused by external, random environmental stresses that are beyond the intent of the design.

2 - **A** = Availability, the probability that a repairable item is in an uptime state with the likelihood of a recoverable downtime state at a particular point in time or during a time interval from t_1 to t_2 (average of the point availabilities). Limiting (steady-state) availability typically occurs at the 6th decimal place when $(\lambda+\mu)*t$ is approximately 10 or more.

3 - **R** = Reliability, the probability of no failures during the time interval from t_1 to t_2. $R \leq 0.3679$ when $N\lambda t \geq 1.0$. The notation $R(t_2 \mid t_1)$, a **conditional probability**, means the "reliability from t_1 to t_2 given the item has operated during the time interval from 0 to t_1 with no failures."

4 - **P** = the probability of r or less number of events (e.g., failures, repairs, etc.) during the time period from 0 to t. This probability is determined by the cumulative Poisson distribution, a series of probability mass functions (pmf). The Poisson process assumes failures are immediately repaired or replaced; there is no accounting for repair time.

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